United Kingdom
Mathematics Trust

Senior Kangaroo

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Organised by the United Kingdom Mathematics Trust

a member of the Association Kangourou sans Frontières

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England & Wales:	*Year* 13 *or below*
Scotland:	*S*6 *or below*
Northern Ireland:	*Year* 14 *or below*

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil** to record your answer to each problem as a three-digit number from 000 to 999.
   Pay close attention to the example on the Answer Sheet that shows how to code your answers.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer;
   There is no penalty for giving an incorrect answer.
7. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Senior Kangaroo should be sent to:

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1. My age is a two-digit number that is a power of 5. My cousin's age is a two-digit number that is a power of 2. The sum of the digits of our ages is an odd number.

What is the product of the digits of our ages?

2. Let K be the largest integer for which $n^{200} < 5^{300}$. What is the value of $10K$?

3. In triangle ABC, we are given that $AC = 5\sqrt{2}$, $BC = 5$ and $\angle BAC = 30°$.

What is the largest possible size in degrees of $\angle ABC$?

4. In a list of five numbers, the first number is 60 and the last number is 300. The product of the first three numbers is 810 000, the product of the three in the middle is 2 430 000 and the product of the last three numbers is 8 100 000.

Which number is third in the list?

5. Rachel and Steven play games of chess. If either wins two consecutive games s/he is declared the champion.

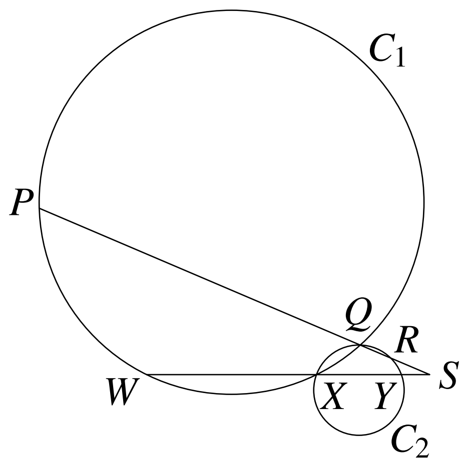
The probability that Rachel will win any given game is 0.6.

The probability that Steven will win any given game is 0.3.

There is a 0.1 probability that any given game is drawn.

The probability that neither is the champion after at most three games is P. Find the value of $1000P$.

6. The line segments $PQRS$ and $WXYS$ intersect circle C_1 at points P, Q, W and X.



The line segments intersect circle C_2 at points Q, R, X and Y. The lengths QR, RS and XY are 7, 9 and 18 respectively. The length WX is six times the length YS.

What is the sum of the lengths of PS and WS?

7. The volume of a cube in cubic metres and its surface area in square metres is numerically equal to four-thirds of the sum of the lengths of its edges in metres.

What is the total volume in cubic metres of twenty-seven such cubes?

8. An integer x satisfies the inequality $x^2 \leq 729 \leq -x^3$. P and Q are possible values of x. What is the maximum possible value of $10(P - Q)$?

9. The two science classes 7A and 7B each consist of a number of boys and a number of girls. Each class has exactly 30 students.

The girls in 7A have a mean score of 48. The overall mean across both classes is 60.

The mean score across all the girls of both classes is also 60.

The 5 girls in 7B have a mean score that is double that of the 15 boys in 7A.

The mean score of the boys in 7B is μ. What is the value of 10μ?

10. The function SPF(n) denotes the sum of the prime factors of n, where the prime factors are not necessarily distinct. For example, $120 = 2^3 \times 3 \times 5$, so SPF(120) $= 2 + 2 + 2 + 3 + 5 = 14$.

Find the value of SPF($2^{22} - 4$).

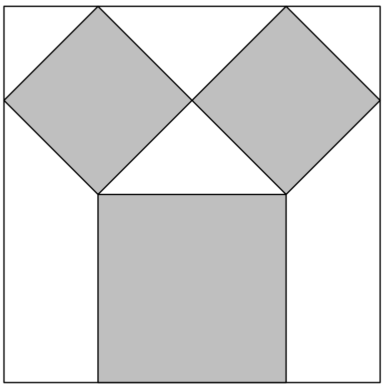
11. A sequence $U_1, U_2, U_3, \ldots$ is defined as follows:

- $U_1 = 2$;
- if U_n is prime then U_{n+1} is the smallest positive integer not yet in the sequence;
- if U_n is not prime then U_{n+1} is the smallest prime not yet in the sequence.

The integer k is the smallest such that $U_{k+1} - U_k > 10$.

What is the value of $k \times U_k$?

12. The diagram shows a 16 metre by 16 metre wall. Three grey squares are painted on the wall as shown.



The two smaller grey squares are equal in size and each makes an angle of 45° with the edge of the wall. The grey squares cover a total area of B metres squared.

What is the value of B?

13. A nine-digit number is odd. The sum of its digits is 10. The product of the digits of the number is non-zero. The number is divisible by seven.

When rounded to three significant figures, how many millions is the number equal to?

14. A square $ABCD$ has side 40 units. Point F is the midpoint of side AD. Point G lies on CF such that $3CG = 2GF$.

What is the area of triangle BCG?

15. In the sequence $20, 18, 2, 20, -18, \ldots$ the first two terms a_1 and a_2 are 20 and 18 respectively. The third term is found by subtracting the second from the first, $a_3 = a_1 - a_2$. The fourth is the sum of the two preceding elements, $a_4 = a_2 + a_3$. Then $a_5 = a_3 - a_4$, $a_6 = a_4 + a_5$, and so on.

What is the sum of the first 2018 terms of this sequence?

16. A right-angled triangle has sides of integer length. One of its sides has length 20. Toni writes down a list of all the different possible hypotenuses of such triangles.

What is the sum of all the numbers in Toni's list?

17. Sarah chooses two numbers a and b from the set $\{1, 2, 3, \ldots, 26\}$. The product ab is equal to the sum of the remaining 24 numbers.

What is the difference between a and b?

18. How many zeros are there at the end of $\dfrac{2018!}{30! \times 11!}$?

19. Shan solves the simultaneous equations

$$xy = 15 \text{ and } (2x - y)^4 = 1$$

where x and y are real numbers. She calculates z, the sum of the squares of all the y-values in her solutions.

What is the value of z?

20. Determine the value of the integer y given that $y = 3x^2$ and

$$\frac{2x}{5} = \cfrac{1}{1 - \cfrac{2}{3 + \cfrac{1}{4 - \cfrac{5}{6 - x}}}}$$