Solutions to the European Kangaroo Pink Paper

1. **C** Dividing 75 km (75 000 m) by 12 500 gives the length of a container as 6 m.

2. **E** If each square has side-length one unit, then the length r is 16 units. The length s consists of 8 straight unit lengths and two semicircles with radius 1 unit, so $s = 8 + 2\pi$. The length t consists of 8 straight unit lengths and two diagonals (that together make the hypotenuse of a right-angled triangle with short sides both of length 4 units), so $t = 8 + 2\sqrt{8}$. Since $\sqrt{8} < 3 < \pi < 4$, we have $t < s < r$.

3. **A** Since $\frac{2}{3} = \frac{10}{15}$ and $\frac{4}{5} = \frac{12}{15}$, the number halfway between $\frac{2}{3}$ and $\frac{4}{5}$ is $\frac{11}{15}$.

4. **C** Working backwards one year at a time, we see that the last digits of 2013, 2012, 2011, 2010 are not larger than the sum of the other digits, but for 2009 the last digit is larger than $2 + 0 + 0$. This was 5 years ago.

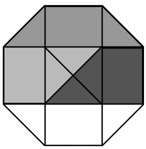
5. **E** The ratio of men to women (2 : 3) is equivalent to 16 : 24; the ratio of women to children (8 : 1) is equivalent to 24 : 3. Hence the ratio of men to women to children is 16 : 24 : 3. Combining men and women gives the ratio of adults to children as 40 : 3.

6. **C** Note that $4.2 = 14 \times 0.3$ and that $0.9 = 3 \times 0.3$. So to determine when the valves are next at their lowest point at the same time we need the lowest common multiple (LCM) of 14 and 3. As these two numbers are coprime, their LCM is their product, that is 42. So the required distance is $42 \times 0.3 \text{ m} = 12.6 \text{ m}$.

7. **C** The three ages are powers of two and also under 100, so must be three of 1, 2, 4, 8, 16, 32, 64. The sum of the first five is only 63, so to make 100, one of them (the grandmother) must be aged 64. This leaves 36 years as the total of the other two ages. The only two that add to 36 are 32 and 4. Hence the granddaughter is four years old, so she was born in 2010.

8. **B** The total time spent in a bathroom is $9 + 11 + 13 + 18 + 22 + 23 = 96$ minutes. Hence they must use at least $\frac{1}{2} \times 96 = 48$ minutes in one of the bathrooms. However, we can show that no combination of these times gives 48 minutes. For one bathroom must take the 23 minute girl and we would need to find other girls' times adding to 25 minutes. This is impossible for any two girls and yet any three girls have a total time greater than 25 minutes. Notice, though, that $11 + 13 = 24$; so we get $11+13 + 23 = 47$ and $9 + 18 + 22 = 49$. Hence they can have breakfast at 7:49am.

9. **D** The original shaded piece can be split into two isosceles right-angled triangles and a rectangle. The remainder of the octagon can be filled with three shapes of area equal to that of the original shaded shape, as shown. The area is then four times the shaded area, namely 12 cm^2.



10. A Since the head of my crocodile is a quarter of the length of it (not counting the tail), this length is 4×93 cm $= 372$ cm. This is also two-thirds of the total length (with the tail as the other third). Hence one-third of the total length is $\frac{1}{2} \times 372$ cm $= 186$ cm, and the total length is 3×186 cm $= 558$ cm.

11. E The only even prime number is 2. The numbers opposite the 18 and 14 are different primes, so at least one of them must be odd. Thus the sum of the opposite pairs must be odd. But then the number opposite 35 must be even and a prime, so it is 2. The sum of opposite pairs is then $35 + 2 = 37$. Hence the number opposite to 14 must be $37 - 14 = 23$.

12. E Let the time for which Ann needs to run be T hours. So the total time for her journey will be $(T + 2)$ hours. In order to have an average speed of 5 km/h over this time she will need to travel $5(T + 2)$ km. After walking 8 km in 2 hours, she will run a distance of $8T$ km if she runs for T hours at 8 km/h. So her total distance travelled will be $(8 + 8T)$ km. Therefore $(8 + 8T) = 5(T + 2)$, that is $8 + 8T = 5T + 10$. So $T = \frac{2}{3}$ and two-thirds of an hour is 40 minutes.

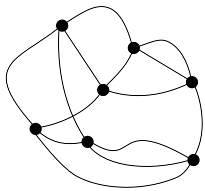
13. C Let W be the number of wins, and L the number of losses. Each win is one point, giving W points for the wins. The number of draws is $40 - W - L$, giving a score of $(40 - W - L)/2$ for the draws. In total the score is 25 points, so we have $W + (40 - W - L)/2 = 25$, leading to $\frac{1}{2}W - \frac{1}{2}L + 20 = 25$, so $\frac{1}{2}W - \frac{1}{2}L = 5$ and $W - L = 10$. Hence the difference between the number of wins and losses is 10.

14. D The total reduction in the price of the three hats is $3 \times 9.40 =$ €28.20. Between them they lack $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{20 + 15 + 12}{60} = \frac{47}{60}$ of the price of a hat. Thus we know that $\frac{47}{60}$ of the price of one hat is €28.20. Thus the price of a hat is $28.20 \times 60 \div 47 =$ €36.

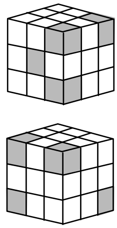
15. C We have $p + \dfrac{1}{q + \frac{1}{r}} = \dfrac{25}{19} = 1 + \dfrac{6}{19}$. Since p, q and r are positive integers, $\dfrac{1}{q + \frac{1}{r}} < 1$. It follows that $p = 1$ and $\dfrac{1}{q + \frac{1}{r}} = \dfrac{6}{19}$. Therefore $q + \dfrac{1}{r} = \dfrac{19}{6} = 3 + \dfrac{1}{6}$. Hence, by a similar argument, $q = 3$ and $r = 6$. Hence $pqr = 1 \times 3 \times 6 = 18$.

16. D The prime factorisation of 33 is 3×11, so apart from a change of order the only way to write 33 as a product of three integers is $1 \times 3 \times 11$. Now $M + B + E + R \geqslant 0 + 1 + 2 + 3 = 6$ so we must have $M + B + E + R = 11$ and N and U are 1 and 3 in either order. The four smallest integers that remain are 0, 2, 4, 5 which sum to 11, so the values of M, B, E, R must in fact be 0, 2, 4, 5 in some order. There are 4 choices for the value of M, leaving three for B, two for E and one for R, giving 24 choices. The total for all the choices is two (for N, U) times 24 (for M, B, E, R), giving 48 choices altogether.

PK14



17. D Let n be the smallest number of connections that each point could have; then the total number of connections from all the points together would be $7n$. Every connecting line has two ends, so contributes two to the number of connections coming from the points. Hence $7n$ must be even, so n must be even. One of the points has 3 connections already, so the smallest possible would be 4 connections from each point. The total number of connections would then be $7 \times 4 = 28$, requiring 14 connecting lines. Subtracting the 5 already there, we would need to add 9 more. This can be achieved as shown in the diagram.



18. D In the two diagrams we can see that the large cube has four white vertices and four grey vertices. Three of the grey vertices lie in the same face; they are in the right hand face of the top diagram, and in the left hand face of the bottom diagram. Hence the lower cube is the upper cube rotated 90° clockwise, as viewed from above. Out of the six cubes in the centres of the faces, we can see that three of them are white, so at most three of them could be grey. Out of the 12 cubes in the middle of the edges, we can see that the top face has no grey ones, the middle layer has no grey ones, and the bottom layer may have one grey cube.

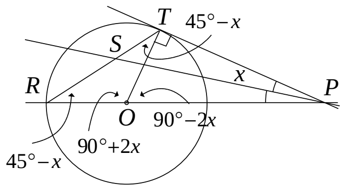
The largest number of grey cubes would therefore arise from four grey cubes at vertices, three in the centres of faces, one in the middle of an edge and the cube in the very centre of the large cube, making a total of nine in all.

19. B Let b be the original number of blue frogs, and g the number of green frogs. The new number of blue frogs is $1.6b$, and the number of green frogs is $0.4g$. The new ratio of blue frogs to green frogs is $1.6b : 0.4g$ and is the same as the previous ratio in the opposite order $g : b$. Hence $\frac{1.6b}{0.4g} = \frac{g}{b}$. This gives $1.6b^2 = 0.4g^2$. which simplifies to $g^2 = 4b^2$ so $g = 2b$. Then the original population of frogs is $b + g = b + 2b = 3b$; and the new population is $1.6b + 0.4g = 1.6b + 0.8b = 2.4b$. This is a reduction of $0.6b$ from the original $3b$, which is a fifth (or 20%).

20. C Since the prime factorisation of 18 is 2×3^2, Tomas must ensure that he does not have any multiple of 2 together with two or more multiples of 3 in the numbers he writes down. If he excludes all multiples of 2, then he writes down 50 numbers. However, if he excludes all but one multiple of 3 (of which there are 33, though the one he includes mustn't be a multiple of 9), then he writes down $100 - 32 = 68$ numbers.

21. C There are eight vertices on a cube. To pick three of these to form a triangle, there are 8 choices for the first vertex, 7 choices for the second vertex, and 6 choices for the third, making $8 \times 7 \times 6 = 336$ choices. However, some of these choices form the same triangles, so we must only count each set of three vertices once. Since each set of three vertices can be arranged in six different ways, we must divide the 336 by 6 to get 56 possible triangles. Now for a particular face, there are 4 possible triangles that can be formed in that face. As there are 6 faces, there are $6 \times 4 = 24$ triangles whose vertices all lie in the same face, and hence $56 - 24 = 32$ triangles whose vertices do not all lie in the same face.

22. B



Denote $\angle SPT$ by x. Since TP is a tangent and OT is a radius, $\angle OTP = 90°$. So $\angle TOP = 180° - \angle OTP - \angle OPT = 180° - 90° - 2x = 90° - 2x$. Then $\angle TOR = 90° + 2x$ (angles on a straight line). But triangle TOR is isosceles (OT and OR are both radii), so $\angle ORT = \angle OTR$. Therefore by considering the angles in the triangle TOR, we have $\angle ORT = \frac{1}{2}(\angle ORT + \angle OTR) = \frac{1}{2}(180° - (90° + 2x)) = 45° - x$. By considering the angles in the triangle TSP, we see $\angle TSP = 180° - \angle SPT - \angle STP = 180° - x - (90° + 45° - x) = 45°$.

23. E The number of integers on Tatiana's list that start with 1, 2 or 3 will be the same as the number of integers that start with 5, 6 or 7. Hence the integers around the middle will all start with the digit 4. Just considering the integers that start with a 4, the number of these whose second digit is 1,2 or 3 will be the same as the number whose second digit is 5, 6 or 7; hence the largest one of the first half of the list will be the largest integer that starts with '43', namely 4 376 521.

24. B Triangle FGH is right-angled with the right angle at H because its sides 6, 8, 10 form a Pythagorean triple. Using the converse of 'angles in a semicircle are right angles', we deduce that FG is the diameter of a circle with centre at I (midpoint of FG) and radius 5 (half of the length FG). Thus IH has length 5 units, and the square $HIJK$ has area $5 \times 5 = 25$. By subtracting the area of triangle HIL we will be able to find the area of quadrilateral $HLJK$ as required. We can find the area of triangle HIL by showing it is similar to triangle FGH: let the angle HFG be x; then the angles in triangle FGH are 90°, x and $90° - x$. Since HI and FI are both 5 units long, triangle HFI is isosceles so we have $\angle IHF = \angle HFG = x$. But then $\angle IHL = 90° - x$, so the angles of triangle HIL are 90°, x and $90° - x$, the same as triangle FGH. Using this similarity $\frac{IL}{IH} = \frac{FH}{HG}$ so $\frac{IL}{5} = \frac{6}{8}$. Hence $IL = \frac{30}{8}$ and area $HIL = \frac{1}{2} \times 5 \times \frac{30}{8} = \frac{75}{8}$. Hence area $HLJK = 25 - \frac{75}{8} = \frac{125}{8}$.

25. C There cannot be more than 1007 knaves, for if there were, then the furthest forward knave would have at least 1007 knaves behind him and at most 1006 knights in front of him, so he would be telling the truth when he says "There are more knaves behind me than knights in front of me". Also, there cannot be more than 1007 knights, for if there were then the furthest back knight would have at least 1007 knights in front of him, and at most 1006 knaves behind him, so he would be lying when he says "There are more knaves behind me than knights in front of me". Hence there must be exactly 1007 knaves, and exactly 1007 knights. This is possible if the 1007 knights stand at the front of the queue, followed by the 1007 knaves.