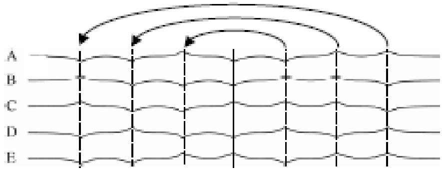
Solutions to the European Kangaroo Pink Paper

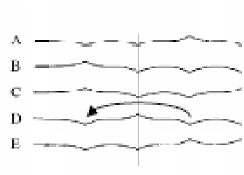
1. **D** $20102010 = 20100000 + 2010 = 2010 \times (10000 + 1) = 2010 \times 10001$.

2. **D** Tibor scored 5% more than Ivan, which is one more mark. Since $100\% = 20 \times 5\%$, Alex scored 20 marks.

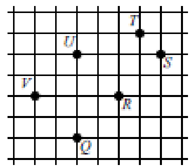
3. **C** Each cube has six identical faces, so the area of each face is $24 \div 6 = 4\ \text{cm}^2$. The cuboid has 16 such faces on its surface so has surface area $16 \times 4 = 64\ \text{cm}^2$.

4. **D** Imagine refolding these strips once, as is shown in the diagram on the left. The peaks on one side of the fold must match with hollows on the other side (which they all do!). We obtain the half-size strips shown in the diagram on the right. Now imagine refolding these strips about their mid-points. We can see that, in D, there are troughs on both sides, so D is not possible; but all the others are possible.





5. **E** The shape *RSUV* is a parallelogram; *RSTU* is a trapezium; *RSU* is a right-angled triangle; *RSV* is an obtuse-angled triangle.
Therefore all the shapes can be made.



6. **C** Brigitte must cross an even number of times so that she returns to the same side of the river as the train station. However, four crossings are not sufficient to cross all five bridges, so the only possibility from the options available is six crossings. Six crossings are possible because she can cross all five bridges, and then return over one of these to the station side.

7. **A** The angles in equilateral triangles are all 60° so $\angle PSU = 90° - 60° = 30°$, $\angle TSU = 30° + 60° = 90°$ and $US = 1 = ST$. Using Pythagoras' theorem on the right-angled triangle *TSU* we have $TU^2 = 1^2 + 1^2 = 2$ so $TU = \sqrt{2}$.

8. **C** The prime factor decomposition of 2010 is $2 \times 3 \times 5 \times 67$ so the product pairs that make 2010 are 1×2010, 2×1005, 3×670, 5×402, 6×335, 10×201, 15×134, 30×67. The only realistic ages for my teacher and his father would be 30 and 67, so my teacher was born 30 years ago, in 1980.

9. **D** Angle *UPT* is $360° - 330° = 30°$ and the reflex angle *QRS* is 270°. Since the angles in the quadrilateral *PQRS* add to 360°, we have $\angle RSP = 360° - (270° + 30° + 20°) = 40°$.

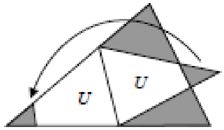
10. **B** Since the product of the digits is 2, there must be a digit 2 somewhere in a 'jumpy' integer, and all the other digits are 1. The digits add to 2010 so there must be exactly 2008 digits that are 1. The digit 2 can be placed before all the ones, after all the ones, or in any of the 2007 places between two ones. Hence there are 2009 'jumpy' integers.

11. B Successive Thursdays are seven days apart, so 'even' Thursdays must be 14 days apart. For there to be three even Thursdays, they must fall on the 2nd, 16th, and 30th days of the month. Hence the 21st day would be five days after a Thursday, which is a Tuesday.

12. C Each of the four congruent parts has three arcs on its perimeter: Two semicircles of radius 2 cm (which have total length $2 \times \pi \times 2 = 4\pi$ cm) and a quarter-arc of radius 4 cm (length $\frac{1}{4} \times 2 \times \pi \times 4 = 2\pi$ cm). Therefore the perimeter has length 6π cm.

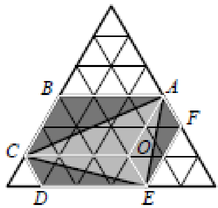
13. D For each runner, the gradient of the line joining the origin to his or her plotted point is equal to the total distance divided by the time taken, which is also his or her average speed. Hence the fastest runner has the steepest line, so it is Dani.

14. B Let U be the unshaded area of the heptagon. Then the area of the triangle is $2U + 1$, as shown in the diagram. This is $1\frac{1}{2}$ times the area of the heptagon, which is $U + 1$, so we can form the equation $2U + 1 = \frac{3}{2}(U + 1)$. So $4U + 2 = 3U + 3$, hence $U = 1$ and the area of the triangle is $2 \times 1 + 1 = 3$.

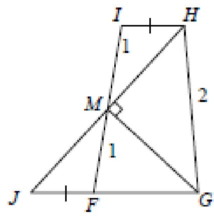


15. C There are 10 more trolleys in the second line, which adds 2 m to the length, so each trolley adds 0.2 m. If we subtract nine of these extra lengths from the first line, we will be left with the length of one trolley, namely $2.9 \text{ m} - 9 \times 0.2 \text{ m} = 1.1 \text{ m}$.

16. A For each of the parallelograms $ABCO$, $CDEO$, $EFAO$ in the diagram, half of its area is from the shaded triangle. Hence the triangle is half of the hexagon formed by the three parallelograms. Since the hexagon is made of 22 triangles, the shaded triangle must have area 11 cm^2.



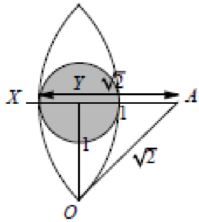
17. B In the diagram, GF and HM are extended to meet at J. Since M is the midpoint of IF, we have $IM = MF$. Also $\angle HMI = \angle FMJ$ (vertically opposite) and $\angle HIM = \angle JFM$ (alternate angles because IH and JF are parallel). Therefore triangles HMI and FMJ are congruent by ASA and in particular $JF = IH$ and also $HM = MJ$.
Also triangles GMJ and GMH are congruent by SAS since they share the side GM, $HM = MJ$, and $\angle GMJ = \angle GMH (= 90°)$. In particular we have $HG = GJ$ so $GJ = 2$. But $GJ = GF + FJ = GF + IH$ so $GF + IH = 2$. The perimeter of the trapezium is therefore $GH + IM + MF + GF + IH = 2 + 1 + 1 + 2 = 6$.



18. B If n is even, say $n = 2m$, then $n^n = n^{2m} = (n^m)^2$ so n^n is a square. There are 50 such n. If n is odd, then n^n cannot be an even power unless n itself is an even power, that is n must be a square. There are five odd squares between 1 and 100 (1, 9, 25, 49, 81). The total number of possibilities for n is $50 + 5 = 55$.

19. B At most one of the statements given by the dragons can be true, so there are at least three liars among them. Since liars have seven legs, these three liars have 21 legs between them. If the fourth dragon is also a liar, they will have 28 legs altogether, meaning that the blue dragon is truthful, causing a contradiction. So the fourth dragon tells the truth, and must have 6 or 8 legs, giving a total number of 27 or 29 legs. The only dragon who could be truthful is the green one who says there are 27 legs. Hence the red dragon is a liar and definitely has 7 legs.

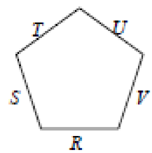
20. A The diagram shows one of the four shaded circles. The point A is a vertex of the original square and O is its centre. So $AY = YO = 1$, and $AX = AO = \sqrt{2}$ by Pythagoras. Also $XY = AX - AY = \sqrt{2} - 1$. So each shaded circle has radius $\sqrt{2} - 1$. Hence the area of the four shaded circles is $4 \times \pi(\sqrt{2}-1)^2 = 4\pi(2-2\sqrt{2}+1) = 4\pi(3-2\sqrt{2})$.



21. A The sequence continues 1, 2, 3, 0, 5, −2, 7, −4, 9, −6, 11, and it can be seen that the terms with even positions are decreasing by 2 (starting with 2). The 2010th term is the 1005th even positioned term, so appears after 1004 decreases. Hence it is $2 - 2 \times 1004 = -2006$.

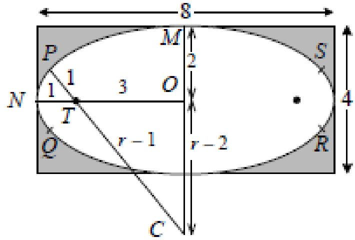
Alternative: We can show this more clearly by considering the terms in pairs. The n th pair has terms $2n - 1$ and $4 - 2n$; this is certainly true for the first two pairs: $2 \times 1 - 1 = 1$ and $4 - 2 \times 1 = 2$ giving the first pair of terms 1, 2, and $2 \times 2 - 1 = 3$ and $4 - 2 \times 2 = 0$, giving the second pair 3, 0. Hence the n th pair and $(n+1)$ th pair will be $2n - 1, 4 - 2n, 2(n+1) - 1, 4 - 2(n+1)$ which simplify to $2n - 1, 4 - 2n, 2n + 1, 2 - 2n$. The rule for finding subsequent terms gives the next pair as $(4-2n) + (2n+1) - (2-2n) = 2n + 3 = 2(n+2) - 1$ and $(2n+1) + (2-2n) - (2n+3) = 4 - 2(n+2)$. These have the same form as $2n - 1$ and $4n - 2$ but with n replaced by $n + 2$. Hence the pattern will continue. The 2010th term is in the 1005th pair, so is $4 - 2 \times 1005 = -2006$.

22. D Let the five numbers be R, S, T, U, V as shown. Then R and T share a common factor greater than 1; so they must also share a common prime factor p, say. Similarly T and V share a common prime factor q, say. But R and V are adjacent so do not share a factor other than 1, meaning that p, q are distinct primes. Therefore T has two distinct prime factors, p and q. This is true for all the edge numbers, but the only option that has two distinct prime factors is $10 = 2 \times 5$. One can check that the five numbers 10, 21, 22, 35, 33 in order, are as required; so 10 is indeed possible.



23. E Let A be the first digit and B the last, then the middle digit is $\frac{1}{2}(A + B)$ which must be a whole number so $A + B$ is even. There are five odd possibilities for A (1, 3, 5, 7, 9), each of which has five possible pairings for B (1, 3, 5, 7, 9), giving 25 possible numbers. There are four even possibilities for A (2, 4, 6, 8), each of which has five possible pairings for B (0, 2, 4, 6, 8), giving 20 possible numbers. Altogether this is 45 possible numbers.

24. A Let C be the centre of the arc PS with radius r, and let T be the centre of the arc PQ. The tangent at P is common to both arcs so the perpendicular at P to this tangent passes through both centres T and C. Let M be the midpoint of the top of the rectangle. The rectangle is tangent to arc PS so the perpendicular from M also passes through C.



Let O be the centre of the rectangle and N the midpoint of the left-hand side. Then $TN = 1$ so $OT = 3$. Also, triangle TCO is right-angled at O with $OT = 3$, $OC = r - 2$ and $CT = r - 1$ so by Pythagoras' Theorem, $(r - 2)^2 + 3^2 = (r - 1)^2$. This gives $r^2 - 4r + 4 + 9 = r^2 - 2r + 1$ so $-4r + 13 = -2r + 1$, leading to $2r = 12$, $r = 6$.

25. D The bar code consists of strips of length one and two, which we can call one-strips and two-strips respectively. Let a be the number of two-strips and b be the number of one-strips. The total length is 12 so $2a + b = 12$. Also, the first and last strips must be black so there is an odd number of alternating strips, meaning $a + b$ is odd.

We know that $a + b$ is odd and $2a + b = 12$ which is even. Therefore $a = (2a + b) - (a + b)$ is odd. Also a is less than 6 since $2a + b = 12$. This gives us three cases:

(i) If $a = 5$, $b = 12 - 10 = 2$.

There are 7 strips altogether. If the first one-strip is the first strip of the bar code then there are 6 options for the position of the second one-strip.

If the first one-strip is the second strip then there are 5 options for the position of the second one-strip, etc. This gives the number of options as $6 + 5 + 4 + 3 + 2 + 1 = 21$.

(ii) If $a = 3$, $b = 12 - 6 = 6$.

If the first two-strip is the first strip, then there are 8 places where the second two-strip can appear, which would leave 7, 6, 5, 4, 3, 2, 1 places for the third two-strip respectively, totalling 28 options.

If the first two-strip is the second strip, then there are 7 places for the second two-strip and 6, 5, 4, 3, 2, 1 places for the third two-strip. Continuing in this way, we see that the total number of options is $28 + 21 + 15 + 10 + 6 + 3 + 1 = 84$.

(iii) If $a = 1$, then $b = 12 - 2 = 10$.

There are 11 strips so the two-strip can appear in 11 places.

In total the number of options is $21 + 84 + 11 = 116$.

Alternatively: the number of ways of choosing the position of 5 two-strips out of 7 is ${}^7C_2 = 21$ and the number of ways of choosing 3 two-strips out of 9 is ${}^9C_3 = 84$.