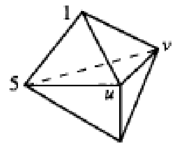
Solutions to the European Kangaroo Pink Paper

1. **C** Kanga divides the other 2008 participants in the ratio 1:3, so has $2008 \div 4 = 502$ participants ahead of her. She comes in 503rd.

2. **A** When multiplying the fractions together, the denominator of each fraction, apart from the last, cancels with the numerator of the next fraction. We are left with the numerator 1 from the first fraction and the denominator 10 from the last, which gives $\frac{1}{10}$ of 1000, i.e. 100.

3. **C** Let the numbers at two of the other vertices be u and v, as shown in the diagram on the right. The three faces sharing the vertex labelled with the number 1 all have the same sum. Then $1 + v + u = 1 + 5 + u$ and so $v = 5$. Similarly, $1 + v + 5 = 1 + v + u$ so $u = 5$. Hence the sum for each face is $1 + 5 + 5$, i.e. 11, and we see that the number at the bottom vertex is 1. The total of all the vertices is $1 + 5 + 5 + 5 + 1 = 17$.

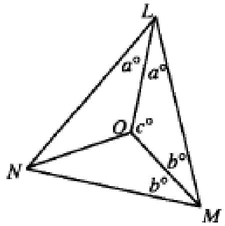


4. **B** Firstly note that if $n \geqslant 10$, then $n^3 \geqslant 10n^2$ so n^3 will have at least one more digit than n^2. For all $n < 10$, we have $n^2 < 100$, so n^2 has either 1 or 2 digits, but n^3 has 3 digits for $n > 4$ since $5^3 = 125$, so we need only consider $n = 1, 2, 3$ or 4. For $n = 1$ and 2, n^2 and n^3 both have one digit; for $n = 4$, $n^2 = 16$ and $n^3 = 64$ both have two digits. However for $n = 3$, $n^2 = 9$ has one digit while $n^3 = 27$ has two digits.

5. **B** The three angles of the triangle add to 180°, so the combined area of the three sectors of the circles that are inside the triangle add up to half a circle with area $\frac{1}{2} \times \pi \times 2^2 = \frac{4\pi}{2} = 2\pi$. So the grey area is $(80 - 2\pi)\,\text{cm}^2$.

6. **E** If we let the fifth number be a, then the sixth number is $6 + a = 15$, so $a = 9$. The seventh number is the sum of the fifth and sixth numbers, $9 + 15 = 24$.

7. **B** Let $\angle OLM = \angle OLN = a°$, $\angle OML = \angle OMN = b°$ and $\angle LOM = c°$. Angles in a triangle add up to 180°, so from ΔLMN, $2a° + 2b° + 68° = 180°$ which gives $2(a° + b°) = 112°$ i.e. $a + b = 56$. Also, from ΔLOM, $a° + b° + c° = 180°$ and so $c = 180 - (a + b) = 180 - 56 = 124$.



8. **E** Mary's average over four tests is 4 marks, so she has scored 16 marks in total. The first four options can give a total of 16 as follows:

A Mary achieved a mark of 4 out of 5 in each test: $4 + 4 + 4 + 4 = 16$

B Mary achieved a mark of 4 out of 5 twice: $4 + 4 + 3 + 5 = 16$

C Mary achieved a mark of 1 out of 5 once: $1 + 5 + 5 + 5 = 16$

D Mary achieved a mark of 3 out of 5 twice: $3 + 3 + 5 + 5 = 16$

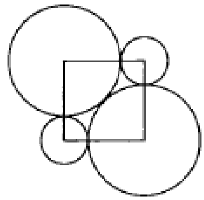
Of the five cases given in the question, only the last one makes it impossible to score 16 marks since she has scored 9 marks from three tests, and would need 7 marks from the fourth test (which is only out of 5).

9. B The first person cannot be telling the truth since if all the others are knaves, this contradicts that they are telling the truth when they say the person in front is a knave!

The second person says the first is a knave so is telling the truth; he is a knight. The third says this knight is a knave so is lying; he is a knave. Continuing in this way we see that there is an alternating sequence of 13 knaves and 12 knights.

10. E $3 \oplus 5 = 3 \times 5 + 3 + 5 = 23$ and $2 \oplus x = 2x + 2 + x = 3x + 2$. These are equal, so $3x + 2 = 23$, i.e. $x = 7$.

11. A Let R be the radius of each of the larger circles. The sides of the square are equal to $R + 1$, the sum of the two radii. The diagonal of the square is $2R$. By Pythagoras, $(R + 1)^2 + (R + 1)^2 = (2R)^2$. Simplifying gives $2(R + 1)^2 = 4R^2$, i.e. $(R + 1)^2 = 2R^2$, so $R + 1 = \sqrt{2}R$ [$-\sqrt{2}R$ is not possible since $R + 1 > 0$]. Therefore $(\sqrt{2} - 1)R = 1$. Hence $R = \dfrac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$.



12. C We require that $9 < \sqrt{n} < 11$, or, equivalently, that $81 < n < 121$. Hence the possible integer values for n are the 39 values $n = 82, 83, \ldots, 119, 120$.

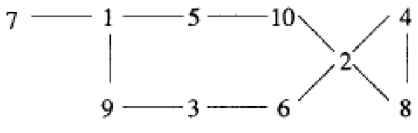
13. D Suppose it is possible to make a list of all ten numbers.

The number 7 must be at one end and must be next to 1 since 7 has no other factors or multiples under 10. Without loss of generality we can assume 7 is the first number, followed by 1.

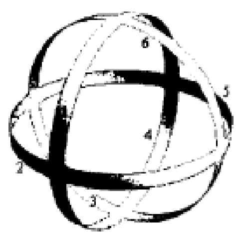
The number 5 only has two possible adjacent numbers, 1 and 10. The same is true for 9 which can only be next to 1 or 3. Hence either we must start with 7, 1, 5, 10 and end with 9; or we start with 7, 1, 9, 3 and end in 5. Either way this means that 1 cannot be next to any other numbers.

The diagram below shows the only possible connections that can be used. It is clearly impossible to link all ten numbers together without using 2 twice. If the sequence starts 7, 1, 5, 10 then the only possibility after 10 is 2 but the only possibility before 6 is 2 which means 2 has to appear twice; or if the sequence starts 7, 1, 9, 3, 6 then the only possibility after 6 is 2 and the only possibility before 10 is 2 so 2 is used twice.

However, the diagram suggests a possible list of nine numbers: 6, 3, 9, 1, 5, 10, 2, 4, 8.



14. A We may suppose that the ant starts at the top. The diagram shows the six quarter-circles that she travels through before arriving back at the top.



15. C The options are all of the form $1 + \frac{1}{n}$ so we need to find n such that $1 + \frac{1}{20008} < 1 + \frac{1}{n} < 1 + \frac{1}{2008}$. This means $2008 < n < 20008$. The choices for n are 100, 1000, 10000, 100000 and 1000000 but only 10000 satisfies the inequalities. We get $1 + \frac{1}{10000} = 1.0001$.

16. E By comparing b and c first, we have $b = 8^8 = (2^3)^8 = 2^{24} = (2^2)^{12} = 4^{12} > 3^{11} = c$ so $c < b$. But also $b = 2^{24} < 2^{25} = a$ so $b < a$. Together these give $c < b < a$.

17. C The digits 1 and 3 will always be followed by the digit 2. The digit 2 can be followed by either 1 or 3. Hence the digit 2 appears exactly five times in a ten-digit number, in alternate positions.

If the first digit is 2, then in each even position we have two choices, 1 or 3. This gives $2 \times 2 \times 2 \times 2 \times 2 = 32$ possibilities. Otherwise, the second digit is 2 and in each odd position we have two choices. So again there are 32 possibilities, making a total of 64.

18. C Any three positive integers that multiply to make 2009 would create viable cuboids. The prime factors of 2009 are $7 \times 7 \times 41$, so the options are $1 \times 1 \times 2009$, $1 \times 7 \times 287$, $1 \times 41 \times 49$ and $7 \times 7 \times 41$. The first three cuboids all have two faces which each require 2009 stickers (1×2009, 7×287 and 41×49 respectively) so Roo cannot cover them. The last cuboid has surface area $2 \times (7 \times 7 + 7 \times 41 + 41 \times 7) = 1246$, leaving $2009 - 1246 = 763$ stickers left over.

19. C Let the smallest prime factor of N be p, whence the second largest factor, and the highest factor Tina wrote down, is $\frac{N}{p}$. Now we have $\frac{N}{p} = 45p$ whence $N = 45p^2$. Since N is a multiple of 45, it has prime factors of 3 and 5 and, because p is the smallest prime factor of N, we can conclude that p can be only 2 or 3. Hence either $N = 45 \times 2^2 = 180$ or $N = 45 \times 3^2 = 405$.

20. B Each fruit can have at most two fruits next to it but each type of fruit must be next to three other types of fruit so there are at least two of every fruit. This means there are at least 8 fruits in total. In fact 8 are sufficient, as shown in the arrangement OABPOBAP (O or Orange, P for Peach, A for Apple, B for Banana).

21. A There are four rows and four columns, so we need eight different sums. The smallest eight sums (if possible) would be 0, 1, 2, 3, …, 7. Since each draught is counted towards the sum of a row and the sum of a column, we would need $\frac{1}{2}(0 + 1 + 2 + \ldots + 7) = 14$ draughts. The diagram shows it is possible to place 14 draughts on the board to create the eight smallest sums (the numbers in the cells represent how many draughts there are in each cell, and the column and row totals are shown).

				0
1	1			2
	1	4		5
	1		6	7
1	3	4	6	

22. B We make use of two key facts. First, we have the factorization $n^2 - 1 = (n-1)(n+1)$. Second, when a square number is factorized, each prime factor appears an even number of times.

Now $2^2 - 1 = 1 \times 3$. We next get a prime factor 3 with $4^2 - 1 = 3 \times 5$. We next get a factor 5 with $6^2 - 1 = 5 \times 7$. We next get a factor 7 with $8^2 - 1 = 7 \times 9$. As $9 = 3^2$, it does not require any further factors. Hence we need $n \geqslant 8$. Checking the product with $n = 8$, we get $(2^2-1)(3^2-1)(4^2-1)(5^2-1)(6^2-1)(7^2-1)(8^2-1) =$

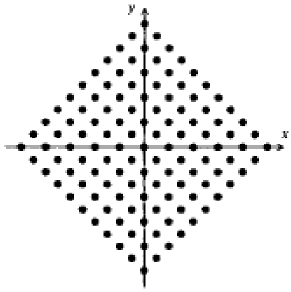
$1 \times 3 \times 2 \times 4 \times 3 \times 5 \times 4 \times 6 \times 5 \times 7 \times 6 \times 8 \times 7 \times 9 =$

$2 \times 8 \times 3 \times 3 \times 4 \times 4 \times 5 \times 5 \times 6 \times 6 \times 7 \times 7 \times 9 =$

$4 \times 4 \times 3 \times 3 \times 4 \times 4 \times 5 \times 5 \times 6 \times 6 \times 7 \times 7 \times 3 \times 3 = (4 \times 3 \times 4 \times 5 \times 6 \times 7 \times 3)^2$.

So in fact $n = 8$ is sufficient, and is thus the minimum.

23. **B** Consider the kangaroo's starting position as the origin of coordinate axes, with East and North being the positive x and y directions, respectively, and one metre being one unit along the axes. We begin by considering the first quadrant. If the kangaroo's end point has coordinates (a, b), then a and b must be integers. Also, after 10 jumps, it must be that $a + b \leqslant 10$. Hence his end points are bounded by the right-angled triangle with vertices at (10, 0), (0, 10) and (0, 0). He can finish at any point on the hypotenuse of this triangle since all these points satisfy $a + b = 10$ and so can be reached by a jumps East and b jumps North. But he can only end up at a point (a, b) on the other two edges or inside the triangle if $a + b$ is even. (He can certainly reach all such points in $a + b \leqslant 10$ jumps, and if $a + b$ is even, with $a + b < 10$, he can jump away and back again using up 2 jumps, and can repeat this until he has made 10 jumps, and so end up at (a, b).)



By symmetry we see that the possible end points form a square of side 11, and so there are 121 of them, as shown in the diagram.

24. **C** The following seven pairs add to 16 so at least one of each pair must be removed: (1, 15), (2, 14), (3, 13), (4, 12), (5, 11), (6, 10), (7, 9).

If removing these seven is sufficient, then we would be left with 8, 16 and seven others.

But	$16 + 9 = 25$	so we must remove 9 (and keep its partner 7).
	$7 + 2 = 9$	so we must remove 2 and keep 14.
	$14 + 11 = 25$	so we must remove 11 and keep 5.
	$5 + 4 = 9$	so we must remove 4 and keep 12.
	$12 + 13 = 25$	so we must remove 13 and keep 3.
	$3 + 1 = 4$	so we must remove 1 and keep 15.
	$15 + 10 = 25$	so we must remove 10 and keep 6.

But we have kept 3 and 6 which add to 9.

Hence it is not sufficient to remove only seven. If we remove the number 6, we obtain a set which satisfies the condition: {8, 16, 7, 14, 5, 12, 3, 15} or in ascending order {3, 5, 7, 8, 12, 14, 15, 16}. Hence eight is the smallest number of numbers that may be removed.

25. **D** The 1-digit primes are 2, 3, 5 and 7. Any two of these make a two-digit number which is strange if the result is a prime. A 2-digit prime cannot end in 2 or 5, and we can also exclude 27, 57 because they are divisible by 3; also 33 and 77 are divisible by 11. This leaves four 2-digit strange primes: 23, 53, 73, 37.

A 3-digit strange prime will be the concatenation of two 2-digit strange primes where the last digit of the first prime is the first digit of the second prime. The possibilities are: 23 and 37 to make 237; 53 and 37 to make 537; 73 and 37 to make 737; 37 and 73 to make 373. However, 237 and 537 are divisible by 3, and 737 is divisible by 11. This leaves only one 3-digit strange prime, 373. Therefore a 4-digit strange prime can only begin with 373 (making the second digit 7) and end with 373 (making the second digit 3) which is impossible. Since there are no 4-digit primes, we cannot make a strange prime with more than 4 digits. Hence there are nine strange primes: 2, 3, 5, 7, 23, 37, 53, 73, 373.