EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 19th March 2009

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 5 million students in over 40 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden.**
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour.* Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think,** not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

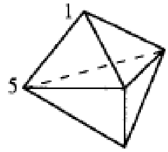
1. The Woomera Marathon had 2009 participants. The number of participants beaten by Kanga was three times the number that beat Kanga. In what position did Kanga finish the marathon?

A 500th B 501st C 503rd D 1503rd E 1507th

2. What is the value of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 1000?

A 100 B 200 C 250 D 300 E none of these

3. The diagram shows a solid with six triangular faces. At each vertex there is a number. Two of the numbers are 1 and 5 as shown. For each face the sum of the numbers at the three vertices of that face is calculated, and all the sums are found to be the same. What is the sum of all five numbers at the vertices?

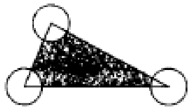


A 9 B 12 C 17 D 18 E 24

4. How many positive integers n exist for which n^2 has the same number of digits as n^3?

A 0 B 3 C 4 D 9 E infinitely many

5. The diagram shows a triangle and three circles whose centres are at the vertices of the triangle. The area of the triangle is 80 cm^2 and each of the circles has radius 2 cm. What is the area, in cm^2, of the shaded area?



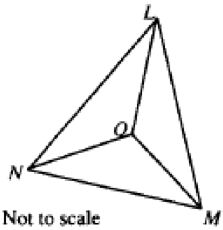
A 76 B $80 - 2\pi$ C $40 - 4\pi$ D $80 - \pi$ E 78π

6. Leonard writes down a sequence of numbers. After the first two numbers, each number is the sum of the previous two numbers in the sequence. The fourth number is six and the sixth number is fifteen. What is the seventh number in the sequence?

A 9 B 16 C 21 D 22 E 24

7. The three angle bisectors of triangle LMN meet at a point O as shown. Angle LNM is 68°. What is the size of angle LOM?

A 120° B 124° C 128° D 132° E 136°



Not to scale

8. Mary sits four tests, each of which is out of 5 marks. Mary's average over the four tests is 4 marks. Which one of the following statements cannot be true?

A Mary obtained a mark of 4 out of 5 in each test
B Mary obtained a mark of 4 out of 5 twice
C Mary obtained a mark of 1 out of 5 once
D Mary obtained a mark of 3 out of 5 twice
E Mary obtained a mark of 3 out of 5 three times

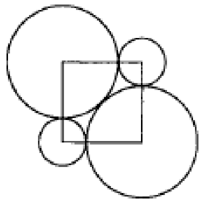
9. A magical island is inhabited entirely by knights (who always tell the truth) and knaves (who always tell lies). One day 25 of the islanders were standing in a queue. The first person in the queue said that everybody behind was a knave. Each of the others in the queue said that the person immediately in front of them in the queue was a knave. How many knights were there in the queue?

A 0 B 12 C 13 D 24 E more information needed

10. We define $a \oplus b = ab + a + b$. Given that $3 \oplus 5 = 2 \oplus x$, what is the value of x ?

A 3 B 4 C 5 D 6 E 7

11. The diagram shows two identical large circles and two identical smaller circles whose centres are at the corners of a square. The two large circles are touching, and they each touch the two smaller circles. The radius of the small circles is 1 cm. What is the radius of a large circle in centimetres?



A $1+\sqrt{2}$ B $\sqrt{5}$ C $\sqrt{2}$ D $\frac{5}{2}$ E $\frac{4}{5}\pi$

12. How many integers n exist such that the difference between $\sqrt{n}$ and 10 is less than 1?

A 19 B 20 C 39 D 40 E 41

13. Peter wishes to write down a list of different positive integers less than or equal to 10 in such a way that for each pair of adjacent numbers one of the numbers is divisible by the other. What is the length of the longest list that Peter could write down?

A 6 B 7 C 8 D 9 E 10

14. Three circular hoops are joined together so that they intersect at right-angles as shown. A ladybird lands on an intersection and crawls around the outside of the hoops by repeating this procedure: she travels along a quarter-circle, turns 90° to the right, travels along a quarter-circle and turns 90° to the left. Proceeding in this way, how many quarter-circles will she travel along before she first returns to her starting point?

A 6 B 9 C 12 D 15 E 18

15. Which of these decimals is less than $\frac{2009}{2008}$ but greater than $\frac{20009}{20008}$?

A 1.01 B 1.001 C 1.0001 D 1.00001 E 1.000001

16. If $a = 2^{25}$, $b = 8^8$ and $c = 3^{11}$, then which of these statements is true?

A $a < b < c$ B $b < a < c$ C $b < c < a$ D $c < a < b$ E $c < b < a$

17. How many ten-digit numbers are there which contain only the digits 1, 2 or 3, and in which any pair of adjacent digits differs by 1?

A 16 B 32 C 64 D 80 E 100

18. Roo has glued 2009 unit cubes together to form a cuboid. He opens a pack containing 2009 stickers and he has enough to place one sticker on each exposed face of each unit cube. How many stickers does he have left?

A fewer than 49 B 49 C 763 D 1246 E more than 1246

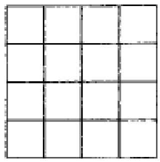
19. When Tina chose a number N and wrote down all of its factors, apart from 1 and N, she noticed that the largest of the factors in the list was 45 times the smallest factor in the list. How many numbers N could Tina have chosen for which this is the case?

A 0 B 1 C 2 D more than 2 E more information needed

20. A grocer places some oranges, peaches, apples and bananas in a row so that, somewhere in the row, each type of fruit can be found next to each other type of fruit. What is the smallest possible number of fruits in the row?

A 7 B 8 C 12 D 16 E 32

21. Barbara wants to place draughts on a 4×4 board in such a way that the number of draughts in each row and in each column are all different (she may place more than one draught in a square, and a square may be empty). What is the smallest number of draughts that she would need?



A 14 B 16 C 21 D 28 E 32

22. What is the smallest integer n such that the product

$$(2^2 - 1)(3^2 - 1)(4^2 - 1)\dots(n^2 - 1)$$

is a perfect square?

A 6 B 8 C 16 D 27 E none of these

23. A kangaroo is sitting in the Australian outback. He plays a game in which he may only jump 1 metre at a time, either North, East, South or West. At how many different points could he end up after 10 jumps?

A 100 B 121 C 400 D 441 E none of these

24. Shakil wants to remove numbers from the set $\{1, 2, 3, \dots, 16\}$ so that no two remaining numbers add to make a perfect square. What is the smallest number of numbers that he needs to remove?

A 6 B 7 C 8 D 9 E 10

25. A prime number is called 'strange' if either it is a one-digit prime, or if each of the numbers obtained by removing its first digit or its last digit are themselves strange primes. How many strange primes are there?

A 6 B 7 C 8 D 9 E 11