Solutions to the European Kangaroo Pink Paper

1. **A** Andy gives away two but gains 4 and ends up with 10, so he must have started with 8.

2. **A** The total number of dots on two dice is $2 \times (1 + 2 + 3 + 4 + 5 + 6) = 42$ and subtracting the visible dots $1 + 2 + 2 + 4 + 6 (= 15)$ leaves 27.

3. **C** The number of members in three years' time will be $32 + 16 = 48$ then $48 + 24 = 72$ and $72 + 36 = 108$.

4. **D** The base *JN* of the new triangle is $\frac{3}{4}$ of the old triangle, and its height is $\frac{1}{2}$ the old height, so its area is $\frac{3}{4} \times \frac{1}{2} \times 96 = 36$.

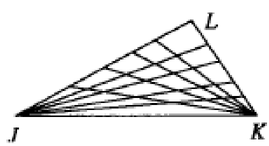
5. **E** The actual number of marbles doesn't matter (so long as the number is divisible by 9). Bag A is left with $\frac{1}{3}$ of its original contents, while Bag C now has $\frac{5}{3}$ so the ratio is 1:5.

6. **A** The table can be filled in just by looking for a row or column with two identical entries already. Notice that the top row has two 0s so the missing entries are both 1. The completed table is

0	1	0	1
1	1	0	0
0	0	1	1
1	0	1	0

7. **B** K and R are hundreds so must be as large as possible, i.e. 9 or 8 (in either order). A, G, O are all in the tens column so they should be 7, 6, 5, but A and O are repeated in the units so should be maximised to 7 and 6 (in either order), leaving G = 5. N must be 4. One possible minimal result is 2007 – 974 – 57 – 866 = 110.

8. **B** The four lines from *J* cut the triangle into five sections. Each of the four lines from *K* cuts five sections, thus creating an extra five sections. Altogether there are $5 + 4 \times 5 (= 25)$ sections.

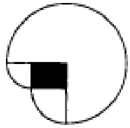


9. **C** The statements are mutually exclusive, and there are knights present so exactly one statement is true. Thus two of the groups must be liars, so there must be at least $2 + 4 = 6$ liars. This means that the first two groups must have been lying, so there are exactly $2 + 4 = 6$ liars.

10. **B** Considering both numbers as powers of 2, we find that $8^8 = (2^3)^8 = 2^{24}$ and $4^4 = (2^2)^4 = 2^8$. Now observe that $2^{24} = (2^8)^3$.

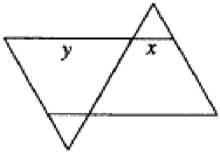
11. **C** Let X be the number of Correct Boys, which is also the number of Incorrect Girls. And let Y be the number of Correct Girls. The total number of correct students is Correct Boys + Correct Girls = X + Y. The number of girls is Correct Girls + Incorrect Girls = Y + X. Hence they are equal.

12. A The dog can trace out $\frac{3}{4}$ of a circle with radius 10 m; $\frac{1}{4}$ of a circle with radius 6 m; and $\frac{1}{4}$ of a circle with radius 4 m. The total perimeter is $\frac{3}{4} \times 2\pi \times 10 + \frac{1}{4} \times 2\pi \times 6 + \frac{1}{4} \times 2\pi \times 4 = 20\pi$.



13. B At his current speed of 100 km/h, Michael has enough fuel to travel 80 km. However, he needs to travel $\frac{3}{4}$ of this distance, so he must travel at $\frac{4}{5}$ of the speed (since his speed and fuel consumption are inversely proportional), i.e. at 80 km/h. At this speed, it will take him $\frac{3}{4}$ of an hour to reach the pump, so he will arrive at 10:15 pm.

14. B Let y be the length of the triangle's edge and let x be the length that is cut off. Then the perimeter of the triangle is $3y$ and the perimeter of the parallelogram is $2(y+x) + 2(y-x) = 4y$. The difference is y which is 10 cm, so the perimeter of the original triangle is 30 cm.



15. E The sequence contains $20 \times 8 = 160$ letters, which we can number 1, 2, 3, etc. The first sweep leaves only the even numbers. The second sweep leaves only multiples of 4. The third sweep leaves only multiples of 8, all of which are the letter O.

16. D There are 10 ways to pick a pair from five players A, B, C, D, E: AB, AC, AD, AE, BC, BD, BE, CD, CE, DE. Each player appears in 4 pairs so he or she must play $4 \times 10 = 40$ games.

17. C Let v be the number of villagers and b be the number of hairs that Barbara has. Now Barbara has more hairs than anyone else and no two people have the same number of hairs so the most villagers there could be is $v = b + 1$ (one for each of the available number of hairs, including zero). But we know $v > b$, so the minimum for v is $b + 1$, hence we must have $v = b + 1$. But we have to use all available numbers of hairs to achieve this, so the gap at 2007 needs to be avoided. Hence $b = 2006$ and $v = 2007$ is the maximum possible.

18. C Along the six edges the centre moves 1 cm (parallel to the edges). Around the six vertices it traces out an arc of radius $\frac{1}{2}$ cm and angle 60°, which has length

$$\frac{60}{360} \times 2\pi \times \frac{1}{2} = \frac{\pi}{6}.$$

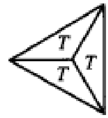
The total distance $= 6 \times \left(1 + \frac{\pi}{6}\right) = 6 + \pi$.

19. C $P(A) = 1 \times \frac{2}{11} \times \frac{1}{10} = \frac{2}{110}$ $\quad$ $P(C) = 1 \times \frac{9}{11} \times \frac{6}{10} = \frac{54}{110}$

$P(B) = 1 \times \frac{8}{11} \times \frac{4}{10} = \frac{32}{110}$ $\quad$ $P(D) = 1 \times \frac{3}{11} \times \frac{2}{10} = \frac{6}{110}$

20. B The total number of diamonds = number of necklaces × number of diamonds per necklace. To be able to know for certain the number of necklaces means that it must be the same as the number of diamonds per necklace and this number must be a prime. Hence the number of diamonds must be the square of a prime. The only candidate between 200 and 300 is 17^2.

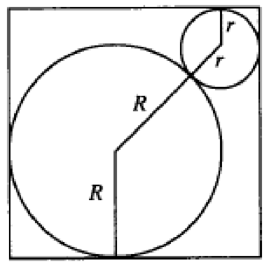
21. E By splitting area S into three small triangles T, we see that $S = 3T$; $L = 12T$; and $H = 6T$. Substituting these into the given expressions, we can see that only E is always true.



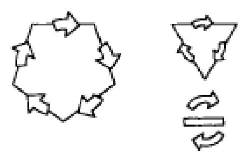
22. D If $10N$ is a square, then N must factorise as $10B^2$ for some integer B. Then $6N = 60B^2$ which factorises as $60B^2 = 2^2 \times 3 \times 5 \times B^2$. We want $6N$ to be a cube, and to be minimal so B must contain 2, 3 and 5 as factors, say $B = 30C$. Then $6N = 2^4 \times 3^3 \times 5^3 \times C^2$ and so $2C^2$ must be a cube. The smallest possible value for C is 2 so the smallest N is $2^5 \times 3^2 \times 5^3$. Now to find the number of factors of N, we can choose from six powers of 2 (including $2^0 = 1$), three powers of 3 and four powers of 5; altogether this is $6 \times 3 \times 4 = 72$ choices.

23. D The three straight lines shown have lengths R, $R + r$ and r. Taking vertical components we have $R + (R + r)\cos 45° + r = 1$. That is $R + \frac{1}{\sqrt{2}}(R + r) + r = 1$ or simply $(R + r)(\sqrt{2} + 1) = \sqrt{2}$, so

$$R + r = \frac{\sqrt{2}}{\sqrt{2} + 1} = 2 - \sqrt{2}.$$



24. C The five presents can be seen as connecting the friends. They either connect them in a ring, or as a pair and a triple. In a ring, the first girl has four choices who she can give to, the second girl has three choices, the next girl two choices, the fourth has to give to the fifth and the fifth has to give to the first girl. This gives $4 \times 3 \times 2 \times 1 = 24$ ways.



There are $5 \times 4 \times 3/3!$ (= 10) choices for those included in the triple. In the triple, the first girl has 2 choices to give to, the second girl has 1 choice. The pair is entirely determined, so there are $10 \times 2 = 20$ choices.

Altogether this is 44 ways.

25. D The tetrahedron could be formed using diagonals on the six faces of a cube with edge 6 cm. The remaining parts of the cube are 4 right-triangle pyramids with volumes $\frac{1}{3} \times \frac{1}{2} \times 6 \times 6 \times 6 = 36$. The cube has volume $6 \times 6 \times 6$ (= 216) so the tetrahedron has volume $216 - 4 \times 36 = 72$.