Solutions to the European Kangaroo Pink Paper

1. **D** The required integer is the mean of 2006 and 6002, which is 4004.

2. **C** The only four-digit multiples of 2006 are 2006, 4012, 6018 and 8024, three of which have four different digits.

3. **D** Arranged in order of size of their first digits, the given numbers are 2, 309, 41, 5, 68 and 7.

4. **E** The required times are 00:26, 02:06, 06:02, 06:20 and 20:06.

5. **E** Half of the upper stripe is shaded, two thirds of the middle stripe and half of the lower stripe. Each stripe is one third of the area of the flag, so the total shaded area is $\left(\frac{1}{2} + \frac{2}{3} + \frac{1}{2}\right) \times \frac{1}{3} = \frac{5}{9}$.

6. **C** Every hour the difference between the times on the watches increases by $1\frac{1}{2}$ minutes, and $60 = 40 \times 1\frac{1}{2}$.

7. **D** We know that each of 25% and $\frac{1}{9}$ of Peter's books is a whole number of books. Therefore the total number of books is a multiple of 4 and of 9, and hence is a multiple of 36. The only such multiple between 50 and 100 is 72.

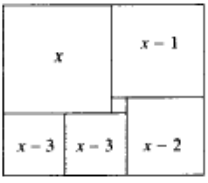
8. **E** The arc of length 2 has an angle of 30° at the centre. Therefore the whole circumference is $2 \times 12 = 24$, since $30° \times 12 = 360°$.

9. **E** Originally, 150 crowns buys 15 packets, including 15 vouchers. For the 15 vouchers you get 5 more packets and 5 more vouchers. For 3 of these vouchers you get 1 more packet and 1 more voucher. You now have 3 vouchers left, so get 1 last packet (and 1 more unusable voucher), making 22 packets in all.

10. **A** $\frac{z}{v} = \frac{wx \times yz}{vw \times xy} = \frac{3 \times 5}{2 \times 4} = \frac{15}{8}$.

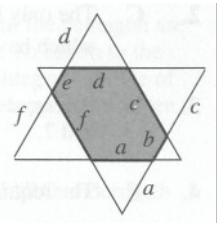
11. **B** Suppose Aunt Bessie was n years old. Then $n = \frac{4}{3} \times \frac{1}{2}(100 - n)$, which gives $3n = 200 - 2n$. Hence $5n = 200$ and so $n = 40$.

12. **D** Let the largest square have side length x. We may deduce the lengths of the sides of the other squares, as indicated, since the smallest square has sides of length 1. Now the width of the rectangle may be found in two different ways, $x + (x - 1)$ or $(x - 3) + (x - 3) + (x - 2)$. Equating these gives $2x - 1 = 3x - 8$ and hence $x = 7$.



13. E From the 'hundreds' and 'thousands' columns we deduce that 3K lies between 18 and 20, so that K = 6. From the 'units' column N is even. The case N = 6 is ruled out since K = 6. The cases N = 0, 2 or 4 each lead to a contradiction, but N = 8 leads to the solution 658 + 659 + 689.

14. B Since the sides of the overlapping triangles are parallel, all the angles in the six smaller triangles surrounding the shaded hexagon are 60°. Hence each of these six triangles is equilateral, and the pairs of sides marked a, c, d and f are equal. Each side of the original triangles has length 6, so that $a + b + c = 6$ and $d + e + f = 6$. Hence the perimeter length of the shaded hexagon, $a + b + c + d + e + f$, equals 12.



15. A The two-digit perfect squares are 16, 25, 36, 49, 64 and 81. Consider pairing these numbers so that the last digit of the first number is equal to the first digit of the second. There are four possible such pairs: (16, 64), (36, 64), (64, 49) and (81, 16). Note that each pair involves a digit 6. The only sequence of digits which can follow the 6 is 4, 9. There are two sequences which can precede it: 3 or 8, 1. So the longest integer as described is 81649.

16. D Choosing three red-blue balls, three blue-green and three green-red shows that 9 balls are not sufficient, since each colour occurs only six times. Choosing 10 balls, however, gives 20 'half balls' of three different colours, so that at least one colour occurs more than six times.

17. E Each part has area 25, so that the four squares have sides of length 5. The whole square has side length $\sqrt{125} = 5\sqrt{5}$, so the length of the shortest side of the L-shaped piece is $5\sqrt{5} - 10$ or $5(\sqrt{5} - 2)$.

18. A The table shows the answers Chris would give to the first question for all four possible combinations of types for Chris, Pat (in that order):

Knight, Knight	Knight, Liar	Liar, Knight	Liar, Liar
Yes	No	Yes	Yes

Since the wise man could not be sure of their types, the single case with answer 'No' is excluded. Now consider the table of Chris's answers to the second question:

Knight, Knight	Knight, Liar	Liar, Knight	Liar, Liar
Yes	–	Yes	No

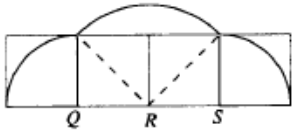
Since the wise man can now identify the types, the single case with answer 'No' is the correct one.

19. B Ignoring the locomotive, there are $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways of arranging the carriages and in half of these ways carriage I will be nearer to the locomotive than carriage II.

20. A Combining the shaded region with the small isosceles right-angled triangle above it forms half a square with area 1/2. The diagonal of the square has length $\sqrt{2}$, so that the small triangle has two sides of length $\sqrt{2} - 1$. Hence the small triangle has area $\frac{1}{2}(\sqrt{2} - 1)^2 = \frac{3}{2} - \sqrt{2}$ and the area of overlap is therefore $\frac{1}{2} - \left(\frac{3}{2} - \sqrt{2}\right) = \sqrt{2} - 1$.

21. C Let the number of children be n. Since the mean age of the whole family is 18 years, the total of their ages is $18(n + 2)$ years. But without including the father the mean is 14, so that the total of all the ages except the father's is $14(n + 1)$ years. Hence $14(n + 1) + 38 = 18(n + 2)$ from which $n = 4$.

22. C The point P moves along three arcs of circles with centres at the places where Q, R and S meet the line, as shown. Each arc is a quarter circle.



The circles have radii 10, $10\sqrt{2}$ and 10, respectively, so that the total length of the curve is $\frac{1}{4} \times 2\pi(10 + 10\sqrt{2} + 10) = 10\pi + 5\pi\sqrt{2}$.

23. B There is one face of each colour. Paint a face with the first colour and place the cube on a table with that face down. The top face may now be painted from a choice of five colours, that is, in 5 ways. Choose one of the remaining colours, paint one of the remaining faces with that colour and turn this face to the front. There remain three faces to colour, which may be done in $3 \times 2 \times 1 = 6$ ways. Therefore there are 5×6 ways altogether.

24. D Consider all the numbers with three distinct digits which also have three digits when reversed. There are nine ways to choose the first digit (from 1 to 9) then 8 ways to choose a different third digit (from 1 to 9) and finally 8 ways to choose a distinct middle digit (from 0 to 9), making $9 \times 8 \times 8$ numbers in all. These numbers may be paired, like 257 and 752, so that one number is the reverse of the other. Since one number in each pair is smaller than the other, exactly half of these 576 numbers have both the required properties.

25. D Represent the scores in the match on a grid as shown.

A goal scored by the home team moves the score one square to the right and an away goal moves it one square up. Since the home team kept the lead until the end, only the positions shown are possible. We need to find the number of ways to move from P (score 1 - 0) to Q (score 5 - 4). Starting from the square next to P, in each square we write the number of ways to move to that square, and deduce there are 14 ways to get to Q.