



**EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'**

Thursday 16th March 2006

**Organised by the United Kingdom Mathematics Trust and the
Association Kangourou Sans Frontières**

This paper is being taken by students in twenty-nine European countries.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
Candidates in Scotland must be in S3 or S4.
Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

*Enquiries about the European Kangaroo should be sent to: Maths Challenges
Office,*

School of Mathematics, University of Leeds, Leeds, LS2 9JT.

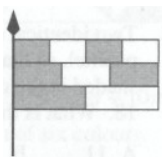
(Tel. 0113 343 2339)

<http://www.ukmt.org.uk>

1. What integer is exactly halfway between 2006 and 6002?
 A 3998 B 4000 C 4002 D 4004 E 4006
2. How many four-digit integers are divisible by 2006 and also have four different digits?
 A 1 B 2 C 3 D 4 E 5
3. What is the smallest 10-digit integer that can be obtained by arranging the following six numbers one after another: 309, 41, 5, 7, 68, and 2?
 A 1 234 567 890 B 1 023 456 789 C 3 097 568 241
 D 2 309 415 687 E 2 309 415 678

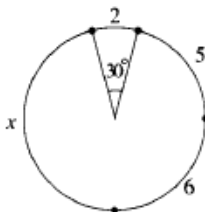
4. How many times does a digital watch show all the four digits 2, 0, 0 and 6, in any order, between 00:00 and 23:59 in the same day?
 A 1 B 2 C 3 D 4 E 5

5. The flag shown in the diagram consists of three stripes, each of equal height, which are divided into two, three and four equal parts, respectively. What fraction of the area of the flag is shaded?



- A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{3}{5}$ D $\frac{4}{7}$ E $\frac{5}{9}$
6. My Grandma's watch gains one minute every hour. My Grandpa's watch loses half a minute every hour. Immediately before leaving their house I set both watches to the correct time and told them 'I will return when the difference between the times on your watches is exactly one hour'. How many hours is it before I return?
 A 12 B $14\frac{1}{2}$ C 40 D 60 E 90
7. Peter says that 25% of his books are novels, and $\frac{1}{9}$ of them are poetry books. Given that Peter has between 50 and 100 books, how many books does he have?

- A 50 B 56 C 64 D 72 E 93
8. The circle shown in the diagram is divided into four arcs of length 2, 5, 6 and x units. The sector with arc length 2 has an angle of 30° at the centre. Determine the value of x .
 A 7 B 8 C 9 D 10 E 11



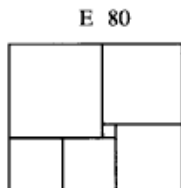
9. One packet of sweets costs 10 crowns. There is a voucher inside every packet. For every three vouchers you collect, you get a free packet of sweets. How many packets of sweets will you get for 150 crowns?
 A 15 B 17 C 20 D 21 E 22
10. Five positive numbers v, w, x, y and z are such that $vw = 2, wx = 3, xy = 4, yz = 5$. What is the value of z/v ?
 A $15/8$ B $5/6$ C $3/2$ D $4/5$ E impossible to determine

11. Simon once asked Aunt Bessie how old she was. Aunt Bessie replied: "If I live to be exactly one hundred, then my age now is four thirds of half of my remaining time." How old was Aunt Bessie at the time?

A 20 B 40 C 50 D 60

12. The rectangle shown is divided into six squares. The length of the sides of the smallest square is 1. What is the length of the sides of the largest square?

A 4 B 5 C 6 D 7 E 8



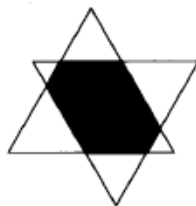
13. Each letter in the sum shown represents a different digit and the digit for A is odd. What digit does G represent?

A 1 B 3 C 5 D 8 E 9

$$\begin{array}{r} \text{K A N} \\ + \text{K A G} \\ + \text{K N G} \\ \hline 2006 \end{array}$$

14. Two identical equilateral triangles overlap with their sides parallel, so that the overlapping region is the hexagon shown shaded in the diagram. The perimeter length of each triangle is 18. What is the perimeter length of the shaded hexagon?

A 11 B 12 C 13 D 14 E 15



15. What is the maximum number of digits that an integer could have if every pair of consecutive digits is a perfect square?

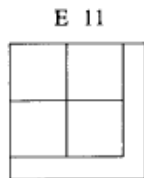
A 5 B 4 C 3 D 6 E 10

16. A box contains 15 balls that are red-blue (half red, half blue), 12 balls that are blue-green and 9 balls that are green-red. What is the smallest number of balls that can be chosen (without looking) to guarantee that at least seven of them share a colour?

A 7 B 8 C 9 D 10

17. A square of area 125 is divided into five parts of equal area – four squares and one L-shaped figure as shown in the picture. What is the length of the shortest side of the L-shaped figure?

A 1 B 1.2 C $2(\sqrt{5} - 2)$
D $3(\sqrt{5} - 1)$ E $5(\sqrt{5} - 2)$



18. A magical island is inhabited by knights (who always tell the truth) and liars (who always lie). A wise man met two people, Chris and Pat, from the island and decided to determine if they were knights or liars. When he asked Chris, "Are you both knights?" he could not be sure of their types. When he then asked Chris, "Are you of the same type?" he could identify their types. What were they?

A both liars B both knights C Chris – knight, Pat – liar
D Chris – liar, Pat – knight E impossible to specify

19. A train consists of five carriages: I, II, III, IV and V. How many ways can the carriages be arranged so that carriage I is nearer to the locomotive than carriage II is?

A 120 B 60 C 48 D 30 E 10

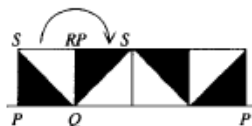
20. Two squares with side 1 have a common vertex, and the edge of one of them lies along the diagonal of the other. What is the area of the overlap between the squares?



- A $\sqrt{2} - 1$ B $\frac{\sqrt{2}}{2}$ C $\frac{\sqrt{2} + 1}{2}$ D $\sqrt{2} + 1$ E $\sqrt{3} - \sqrt{2}$
21. The Dobson family consists of the father, the mother, and some children. The mean age of the Dobson family is 18 years. Without the 38-year-old father the mean age of the family decreases to only 14 years. How many children are there in the Dobson family?

A 2 B 3 C 4 D 5 E 6

22. A square $PQRS$ with sides of length 10 is rolled without slipping along a line. Initially P and Q are on the line and the first roll is around point Q as shown in the diagram. The rolling stops when P first returns to the line. What is the length of the curve that P has travelled?



- A 10π B $5\pi + 5\pi\sqrt{2}$ C $10\pi + 5\pi\sqrt{2}$ D $5\pi + 10\pi\sqrt{2}$ E $10\pi + 10\pi\sqrt{2}$
23. Each face of a cube is painted with a different colour from a selection of six colours. How many different cubes can be made in this way?
- A 24 B 30 C 36 D 42 E 48
24. The number 257 has 3 distinct digits and creates the bigger number 752 when its digits are reversed. How many 3-digit integers have both of these properties?
- A 124 B 252 C 280 D 288 E 360
25. Suppose the final result of a football match is 5 – 4 to the home team. The home team scored first and kept the lead until the end. In how many different orders could the goals have been scored?
- A 17 B 13 C 20 D 14 E 9