Solutions to the European Kangaroo Pink Paper

1. **C** Tidying leaves $-1 - (-1) - (-1) - (-1) - (-1) - (-1) = 4$.

2. **A** $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{11}{12}$ which leaves $\frac{1}{12}$ of 2004, that is 167 marbles.

3. **C** With 7 faces the pyramid must have a hexagonal base, so there are 6 + 6 edges.

4. **E** The floor has a perimeter of $2 \times (40 + 60)$ m $= 20\,000$ cm so the ratio of floor : plan is 100 : 20 000 which simplifies to 1:200.

5. **D** Let Timmy have x points and Tommy have y points. From the given information, $x + 5 = 2y$ and $x - y = \frac{1}{2}y$. Multiplying the second equation by 4 and equating the two expressions for $2y$ gives $x + 5 = 4(x - y)$ whence $3x = 33$ and $x = 11$. Hence Timmy has 11 points.

6. **D** Since $\angle QPR = 180° - (75° + 30°) = 75°$, triangle PQR is isosceles with $QR = PR = PS$. Hence triangle PRS is isosceles so that $\angle PSR = \frac{1}{2}(180° - 50°) = 65°$.

7. **C** When 12 apples are removed there is at least one Granny Smith amongst them, so there are at most 11 Red Delicious apples.
When 20 apples are removed there is at least one Red Delicious amongst them, so there are at most 19 Granny Smith apples.

8. **A** In the large board there would be 2003 + 2002 shaded squares. If instead this number of squares were shaded along two adjacent edges, the white region would then be a square of side 2002.

9. **D** Let the radius of the inner circle be R so its area is then πR^2. The white ring and the inner black circle together have area $\pi(2R)^2 = 4\pi R^2$. The whole target has area $\pi(3R)^2 = 9\pi R^2$. Hence the outer black circle has area $9\pi R^2 - 4\pi R^2 = 5\pi R^2$ which is 5 times the area of the inner circle.

10. **B** The ratio of nuts that Cedric and Celia each take is 3 : 4 whilst the ratio for Cecily and Celia is 7 : 6. So for every 9 nuts Cedric gets, Celia gets 12 nuts and Cecily gets 14 nuts. Now $9 + 12 + 14 = 35$ and $770 \div 35 = 22$ so the youngest squirrel (Cedric) gets $9 \times 22 = 198$ nuts.

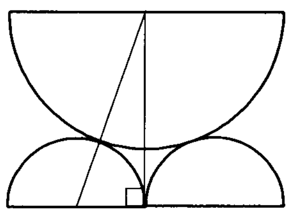
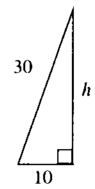
11. **B** The product will be a power of 2 and the largest possible value is $4 \times 4 \times 4 \times 4 \times 4 = 1024$. The only number given which is a power of 2 and at most 1024 is 256.

12. **D** The diameter of each ring is 6 cm. Each extra ring adds a further 4 cm to the length of the chain. If there are n rings, then $6 + 4(n - 1) = 170$, which gives $n = 42$.

13. C The water has base of area 200 cm^2 and volume 1000 cm^3. The empty tank has base of area 100 cm^2 and volume 700 cm^3. The water displaced by putting the empty tank in Tank X is then 700 cm^3. The water in the empty tank is $1000 - 700 = 300\,\text{cm}^3$ and as the base area is 100 cm^2 the water will have depth 3 cm.

14. E Note that the ratio is 1 : 24 whatever the time period, since the minute hand turns 12 times as fast as the hour hand and is twice as long.

15. B Drawing in the triangle as shown we have
$h^2 = 30^2 - 10^2 = 900 - 100 = 800$
and $h = \sqrt{800} = 20\sqrt{2}$.

16. D Suppose Kanga answers c questions correctly and w questions wrongly. Then $7c - 2w = 87$ or $7c = 87 + 2w$, and so $87 + 2w$ is a multiple of 7. The first multiple of 7 above 87 is 91, giving $c = 13$, $w = 2$ and 5 questions missed out. The next possible multiple of 7 is 105, giving $c = 15$, $w = 9$, which is impossible since there are only 20 questions. Clearly no higher values are possible.

17. D The only numbers between 100 and 200 which have 2 and 3 as their only prime factors are $2^7 = 128$, $2^6 \times 3 = 192$, $2^4 \times 3^2 = 144$, $2^2 \times 3^3 = 108$ and $2 \times 3^4 = 162$.

18. C The left-hand two columns can only be completed in one way as shown.

B R *R B*	1
Y G *G Y*	2

Then the square labelled 1 can only be $\begin{smallmatrix} Y & G \\ G & Y \end{smallmatrix}$ or $\begin{smallmatrix} G & Y \\ Y & G \end{smallmatrix}$ and the square labelled 2 can only be $\begin{smallmatrix} R & B \\ B & R \end{smallmatrix}$ or $\begin{smallmatrix} B & R \\ R & B \end{smallmatrix}$. All combinations are possible, giving 2×2 ways in total.

19. C Andrew turned back three times in total as marked on the right-hand diagram.

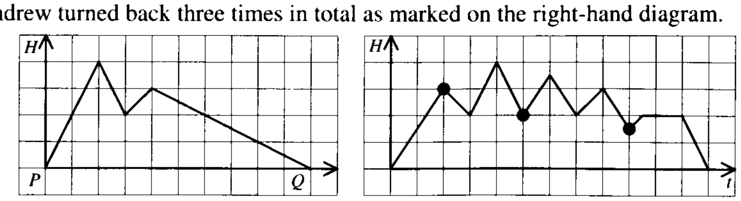
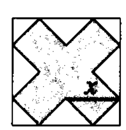


Figure 1 Figure 2

20. B Each side of the dodecagon is of length $36 \div 12$ cm $= 3$ cm. Calculating the marked length gives $x = \sqrt{18}$. The square has side of length $2\sqrt{18}$ cm and area $(2\sqrt{18})^2$ cm^2 $= 72$ cm^2.

OR The dodecagon can be divided into 5 squares of area 9 cm^2. The white triangles can be reassembled into 3 equal squares making 8 in total. So the total area is 8×9 cm^2 $= 72$ cm^2.

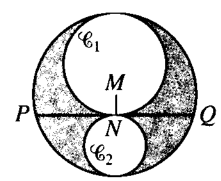


21. E Since 7 is prime, exactly one of $n + 1$, $n + 2$ and $n + 3$ can be a multiple of 7, between 101 and 203. The possible multiples of 7 are 105, 112, … , 203, that is, 15 numbers. All of these except 203 are possible values of $n + 1$, $n + 2$ or $n + 3$, but 203 can only be a value of $n + 3$ since $n \leqslant 200$. So there are $14 \times 3 + 1$, that is, 43 possible values of n.

22. B The number in the 120th position is increased by 1 during each round corresponding to one of the factors of 120. Since 120 has 16 factors, namely 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60 and 120, the number in the 120th position after 200 rounds is 16.

23. B If $a + c + e + g + i = 1$ there are 5 possibilities for $bdfhj$. When $a + c + e + g + i = 2$ there are 4 possibilities for $cegi$ and 10 possibilities for $bdfhj$ giving 40 in total. When $a + c + e + g + i = 3$ there are 6 possibilities for $cegi$ and 10 possibilities for $bdfhi$ giving 60 in total. When $a + c + e + g + i = 4$ there are 4 possibilities for $cegi$ and 5 possibilities for $bdfhj$ giving 20 in total. When $a + c + e + g + i = 5$ there is only 1 possibility for $cegi$ and 1 possibility for $bdfhj$ giving a total of 1. Hence the overall total is $5 + 40 + 60 + 20 + 1 = 126$.

24. D



Let the radius of circle $\mathscr{C}_1$ be R and the radius of circle $\mathscr{C}_2$ be r. Then the shaded shape has area $\pi(R + r)^2 - \pi R^2 - \pi r^2$. Since we are told the area is 2π, $Rr = 1$.

If M is the centre of the outer circle the triangle PMN is right-angled at N and the line MN bisects PQ. Further $PM = R + r$ and $MN = R - r$, so Pythagoras' Theorem gives

$$PN^2 = (R + r)^2 - (R - r)^2 = 4Rr.$$

So $PN^2 = 4 \times 1 = 4$ and $PN = \sqrt{4} = 2$. Hence $PQ = 4$.

25. D In the block of numbers 1, 2, …, 55, there are 5 multiples of 11 and 11 multiples of 5, including 55 which is a multiple of both. So Owl leaves 15 of these numbers on the blackboard, namely 5, 10, 11, 15, 20, 22, 25, 30, 33, 35, 40, 44, 45, 50, 55. The same applies to all subsequent blocks of 55 numbers. Now $2004 = 15 \times 133 + 9$, so the 2004th term is the 9th number following 55×133, that is, $7315 + 33$.