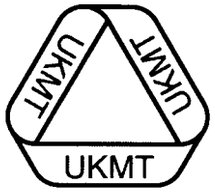


EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 20th March 2003

Organised by the United Kingdom Mathematics Trust and the Association Kangourou des Mathématiques, Paris

This paper is being taken by students in twenty-six European countries.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of **calculators**, rulers and measuring instruments is **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour.* Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)

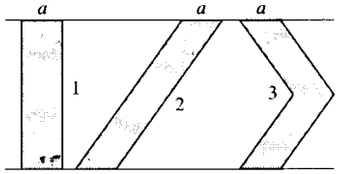
1. One slice of a circular cake is 15 % of the whole cake. What is the size of the angle marked with the question mark?

   A 30° B 45° C 54° D 15° E 20°

2. A circular flowerbed in Kanga's garden has a diameter of 1.2 m. At a nearby park there is a circular flowerbed whose area is four times the size of the one in Kanga's garden. What is the diameter of the flowerbed in the park?

   A 2.4 m B 3.6 m C 4.8 m D 6.4 m E 9.6 m

3. In the picture, the three strips labelled 1, 2, 3 have the same horizontal width a. These three strips connect two parallel lines. Which of these statements is true?



   A All three strips have the same area.
   B Strip 1 has the largest area.
   C Strip 2 has the largest area.
   D Strip 3 has the largest area.
   E It is impossible to say which has the largest area without knowing a.

4. Which of the following numbers is odd for every integer n?

   A $2003n$ B $n^2 + 2003$ C n^3 D $n + 2004$ E $2n^2 + 2003$

5. In a triangle ABC the angle C is three times the size of angle A and the angle B is twice the size of angle A. Then the triangle ABC

   A is equilateral B is isosceles C has an obtuse angle
   D has a right-angle E has only acute angles

6. Jenny, Rachel and Angela sing a song which consists of three equal lines. Rachel starts to sing as Jenny is starting the second line. Angela starts singing as Jenny is starting the third line. Each person sings the whole song four times without a break and then stops. The fraction of the total singing time that all three are singing at the same time is

   A $\frac{3}{5}$ B $\frac{4}{5}$ C $\frac{4}{7}$ D $\frac{5}{7}$ E $\frac{7}{11}$

7. A is the number 11111...1111 formed with all 2003 digits equal to 1. What is the sum of the digits of the product $2003 \times A$?

   A 10000 B 10015 C 10020 D 10030 E 2003×2003

8. In figure 1, alongside, the area of the square equals a. The area of each circle in both figures equals b. Three circles are lined up as shown in figure 2. An elastic band is placed around these three circles without moving them. What is the area inside the elastic band?


Figure 1

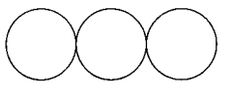
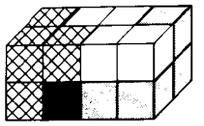


Figure 2

   A $3b$ B $2a + b$ C $a + 2b$ D $3a$ E $a + b$

9. The cuboid shown has been built using four shapes, each made from four small cubes. Three of the shapes can be completely seen, but the dark one is only partly visible. Which of the following shapes could be the dark one?



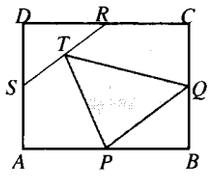
A  B  C  D  E 

10. In the sum on the right, each of the letters X, Y and Z represents a different non-zero digit. What does X represent?

$$\begin{array}{r} XX \\ YY \\ +\,ZZ \\ \hline ZYX \end{array}$$

A 1 B 2 C 7 D 8 E 9

11. In the rectangle $ABCD$, let P, Q, R and S be the midpoints of sides AB, BC, CD and AD, respectively, and let T be the midpoint of segment RS. Which fraction of the area of $ABCD$ does triangle PQT cover?



A $\frac{5}{16}$ B $\frac{1}{4}$ C $\frac{1}{5}$ D $\frac{1}{6}$ E $\frac{3}{8}$

12. Kanga hops to the grazing land and back in 15 minutes. Her speed on the way to the grazing land is 5 m/s, and on the way back her speed is 4 m/s. The distance to the grazing land is:

A 4.05 km B 8.1 km C 0.9 km D 2 km E impossible to determine

13. When a barrel is 30% empty it contains 30 litres more than when it is 30% full. How many litres does the barrel hold when full?

A 60 B 75 C 90 D 100 E 120

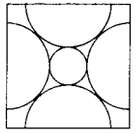
14. Andrew and Bob each start with the 3-digit number 888 which is clearly divisible by 8. Andrew changes two of its digits in order to get as large a 3-digit number as he can which is still divisible by 8. Bob also changes two of the digits of 888 in order to get as small a 3-digit number as he can which is still divisible by 8. What is the difference between their two results? (None of the 3-digit numbers is allowed to begin with 0.)

A 800 B 840 C 856 D 864 E 904

15. The value of the expression $\left(1+\frac{1}{2}\right)\times\left(1+\frac{1}{3}\right)\times\ldots\left(1+\frac{1}{2003}\right)$ is equal to

A 2004 B 2003 C 2002 D 1002 E 1001

16. The diagram shows four semicircles with radius 1. The centres of the semicircles are at the mid-points of the sides of a square. What is the radius of the circle which touches all four semicircles?



A $\sqrt{2}-1$ B $\frac{1}{2}\pi-1$ C $\sqrt{3}-1$ D $\sqrt{5}-2$ E $\sqrt{7}-2$

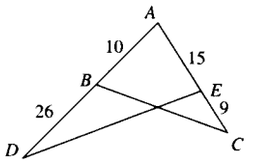
17. One rainy day Kanga set Roo the following problem: 'Add together all the different four-digit numbers that can be made from the digits of 2003 exactly once'. (None of these numbers start with 0.) Which of the following answers should Roo get?

A 5005 B 5555 C 16665 D 1110 E 15555

18. The first two terms of a sequence are 1 and 2. Each new term is obtained by dividing the term two before by the term one before. For example the third term is $1 \div 2$. What is the tenth term of this sequence?

A 2^{-10} B 256 C 2^{-13} D 1024 E 2^{34}

19. What is the ratio of the areas of the triangles ADE and ABC in the picture?



A 9 : 4 B 7 : 3 C 4 : 5 D 15 : 10 E 26 : 9

20. The children P, Q, R and S made the following assertions.

P said: Q, R and S are girls
Q said: P, R and S are boys
R said: P and Q are lying
S said: P, Q and R are telling the truth

How many of the children were telling the truth?

A 0 B 1 C 2 D 3 E It cannot be determined

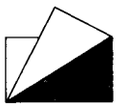
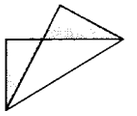
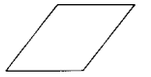
21.   

Diagram A Diagram B Diagram C

A rectangular sheet of paper which measures 6 cm × 12 cm is folded along its diagonal (Diagram A). The shaded areas in Diagram B are then cut off and the paper is unfolded leaving the rhombus shown in Diagram C. What is the length of the side of the rhombus?

A $\frac{7}{2}\sqrt{5}$ cm B 7.35 cm C 7.5 cm D 7.85 cm E 8.1 cm

22. The number of different pairs of integers (x, y), not necessarily positive, which satisfy the equation $(x+y)^2 = (x+3)(y-3)$ is

A 0 B 1 C 2 D 3 E infinitely many

23. What is the greatest number of consecutive positive integers, none of which has the sum of its digits divisible by 5?

A 5 B 6 C 7 D 8 E 9

24. Three different numbers, a, b, c, are chosen from the set $\{1, 4, 7, 10, 13, 16, 19, 22, 25, 28\}$. How many different answers for $a+b+c$ are there?

A 13 B 21 C 22 D 30 E 120

25. Kanga wrote a list of integers which used the digits 1 and 0 only. If she listed every possible integer with at least one digit and no more than 7 digits, how many times did she use the digit 1?

A 128 B 288 C 448 D 512 E 896