The United Kingdom Mathematics Trust

Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Hamilton Paper

Thursday 17th March 2011

All candidates must be in *School Year 10* (England and Wales), *S3* (Scotland), or *School Year 11* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.
   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

   Do not hand in rough work.
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.
   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.
Enquiries should be sent to: Maths Challenges Office,
School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

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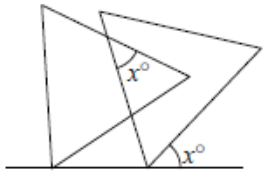
1. If Julie gave £12 to her brother Garron, then he would have half the amount that she would have. If instead Garron gave £12 to his sister Julie, then she would have three times the amount that he would have.

How much money do they each have?

2. The diagram shows two equilateral triangles. The angles marked x° are equal.

Prove that $x > 30$.



3. A particular four-digit number N is such that:

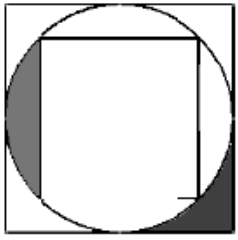
(a) the sum of N and 74 is a square; and

(b) the difference between N and 15 is also a square.

What is the number N?

4. A square just fits within a circle, which itself just fits within another square, as shown in the diagram.

Find the ratio of the two shaded areas.



5. In how many distinct ways can a cubical die be numbered from 1 to 6 so that consecutive numbers are on adjacent faces? Numberings that are obtained from each other by rotation or reflection are considered indistinguishable.

6. Sam wishes to place all the numbers from 1 to 10 in the circles, one to each circle, so that each line of three circles has the same total.

Prove that Sam's task is impossible.