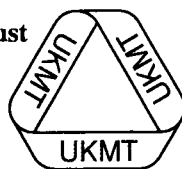


The United Kingdom Mathematics Trust



## Intermediate Mathematical Olympiad and Kangaroo (IMOK)

### Olympiad Cayley/Hamilton/Maclaurin Papers

Thursday 13th March 2008

#### READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**  
Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.  
You may wish to work in rough first, then set out your final solution with clear explanations and proofs.  
***Do not hand in rough work.***
5. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like  $\pi$ , fractions, or square roots if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.  
Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.  
Do not hurry, but spend time working carefully on one question before attempting another. Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

#### DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.

*Enquiries should be sent to: Maths Challenges Office,*

*School of Mathematics, University of Leeds, Leeds, LS2 9JT.*

*(Tel. 0113 343 2339)*

*<http://www.ukmt.org.uk>*

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- *Try to finish whole questions even if you cannot do many.*
- *You will have done well if you hand in full solutions to two or more questions.*
- *Answers must be FULLY SIMPLIFIED, and EXACT using symbols like  $\pi$ , fractions, or square roots if appropriate, but NOT decimal approximations.*
- *Give full written solutions, including mathematical reasons as to why your method is correct.*
- *Just stating an answer, even a correct one, will earn you very few marks.*
- *Incomplete or poorly presented solutions will not receive full marks.*
  
- *Do not hand in rough work*

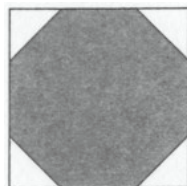
## Olympiad Hamilton Paper

All candidates must be in *School Year 10 (England and Wales), S3 (Scotland), or School Year 11 (Northern Ireland)*.

1. How many four-digit multiples of 9 consist of four different odd digits?

2. A regular octagon with sides of length  $a$  is inscribed in a square with sides of length 1, as shown.

Prove that  $a^2 + 2a = 1$ .



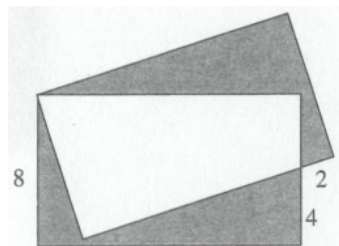
3. Kelly cycles to a friend's house at an average speed of 12 km/hr. Her friend is out, so Kelly immediately returns home by the same route. At what average speed does she need to cycle home if her average speed over the whole journey is to be 15 km/hr?

4. A triangle is bounded by the lines whose equations are  $y = -x - 1$ ,  $y = 2x - 1$  and  $y = k$ , where  $k$  is a positive integer.

For what values of  $k$  is the area of the triangle less than 2008?

5. Two congruent rectangles have a common vertex and overlap as shown in the diagram.

What is the total shaded area?



6. Find all solutions to the simultaneous equations

$$x^2 - y^2 = -5$$

$$2x^2 + xy - y^2 = 5.$$