The United Kingdom Mathematics Trust

Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Hamilton Paper

Thursday 15th March 2007

All candidates must be in *School Year 10* (England and Wales), *S3* (Scotland), or *School Year 11* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.
   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.
   Do not hand in rough work.
5. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.
   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.
Enquiries should be sent to: Maths Challenges Office,
School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

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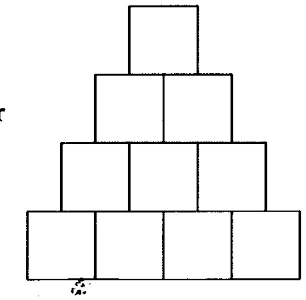
- ***Do not hand in rough work***

1. Numbers are placed in the blocks shown alongside according to the following two rules.

(a) For two adjacent blocks in the bottom row, the number in the block to the right is twice the number in the block to the left.

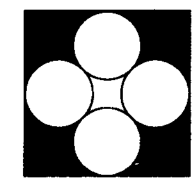
(b) The number in a block above the bottom row is the sum of the numbers in the two adjacent blocks immediately below it.

What is the smallest positive integer that can be placed in the bottom left-hand block so that the sum of all ten numbers is a cube?



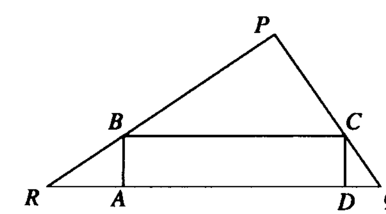
2. Prove that there is exactly one sequence of five consecutive positive integers in which the sum of the squares of the first three integers is equal to the sum of the squares of the other two integers.

3. The diagram shows four circles of radius 1 placed inside a square so that they are tangential to the sides of the square at the midpoints of the sides, and to each other. Calculate the shaded area.



4. Inzamam runs twice as fast as he walks. On Monday, when going to school, he walked for twice the time for which he ran. On Tuesday, doing the same journey, he ran for twice the time that he walked and was six minutes quicker than on Monday. On Wednesday, he walked all the way to school. How long did it take him?

5. The diagram shows a rectangle *ABCD* inscribed inside a triangle *PQR*. The side, *AB*, of the rectangle is one third of the perpendicular height of the triangle from *P* to *QR*.
What is the ratio of the area of the rectangle to the area of the triangle?



6. The numbers 1 to 10 are to be placed in the unshaded boxes, so that the two rows of four boxes and the two columns of three boxes all have the same total T.

(a) Find a solution when $T = 20$.

(b) Find the minimum possible value of T.