

Olympiad Hamilton Paper

All candidates must be in *School Year 10* (England and Wales), *S3* (Scotland), or *School Year 11* (Northern Ireland).

1. (a) A positive integer N is written using only the digits 2 and 3, with each appearing at least once. If N is divisible by 2 and by 3, what is the smallest possible integer N ?
- (b) A positive integer M is written using only the digits 8 and 9, with each appearing at least once. If M is divisible by 8 and by 9, what is the smallest possible integer M ?

2. Triangle ABG has a right angle at B .

Points C and E lie on side AG and points D and F lie on side BG so that the six line segments AB, BC, CD, DE, EF and FG are equal in length. Calculate the angle AGB .

3. An $s \times s$ square, where s is an odd integer, is divided into unit squares (1×1). All the unit squares along the edges and the two diagonals of the $s \times s$ square are discarded. Find a fully simplified expression, in terms of s , for the number of unit squares remaining.
4. The first two terms of a sequence are the numbers 1, 2. From then on, each term is obtained by adding 1 to the previous term and then dividing by the term before that. Thus the third term is obtained by adding 1 to the second term and then dividing by the first term.
 - (a) Write down the first five terms.
 - (b) Calculate the sixtieth term.
 - (c) What happens if other non-zero numbers are chosen for the first two terms, but the rule for calculating the next term remains the same?

5. (a) What is the angle A between the hands of a clock at two o'clock?
- (b) What is the next time after this that the angle between the hands is equal to A ?



6. The triangle ABC is right-angled at A , with $AB = 6$ cm and $AC = 8$ cm. Points X and Y are situated on BC such that $AB = AY$ and $AX = AC$. Two isosceles triangles ABY and AXC are thus created. These triangles overlap, forming the region AXY . Calculate the area of this region.