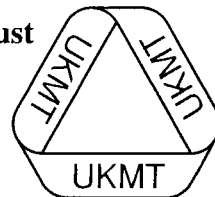


The United Kingdom Mathematics Trust



**Intermediate Mathematical Olympiad and Kangaroo  
(IMOK)**

**Olympiad Hamilton Paper**

Thursday 20th March 2003

All candidates must be in *School Year 10* (England and Wales), *S3* (Scotland), or *School Year 11* (Northern Ireland).

**READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING**

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**  
Rulers and compasses may be used.
3. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Cover Sheet.

For questions in Section B start each question on a fresh A4 sheet and give *full written solutions*, including clear mathematical explanations as to why your method is correct.

Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.

***Do not hand in rough work.***

4. Questions A1-A5 are relatively short questions. Try to complete Section A within the first 20 minutes so as to allow sufficient time for Section B.
5. Questions B1-B5 are longer questions requiring *full written solutions*. This means that each answer must be accompanied by clear explanations and proofs. Work in rough first, then set out your final solution with clear explanations of each step.
6. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
7. Numerical answers must be FULLY SIMPLIFIED, and EXACT using symbols like  $\pi$ , fractions, or square roots if appropriate, but NOT decimal approximations.

**DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!**

## Section A

Write your answers in the boxes provided on the Cover Sheet. Do not hand in your working.  
You should aim to spend no more than 20 minutes on this section.

**A1**  $A$  is the number 11111...1111 formed with all 2003 digits equal to 1. What is the sum of the digits of the product  $2003 \times A$ ?

**A2** In figure 1, alongside, the area of the square equals  $a$ . The area of each circle in both figures equals  $b$ . Three circles are lined up as shown in figure 2. An elastic band is stretched around these three circles without moving them. What is the area inside the elastic band?



Figure 1

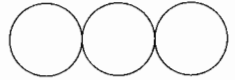


Figure 2

**A3** In the sum on the right, each of the letters  $X$ ,  $Y$  and  $Z$  represents a different non-zero digit. What does  $X$  represent?

$$\begin{array}{r} XX \\ YY \\ + ZZ \\ \hline ZYX \end{array}$$

**A4** Kanga hops to the grazing land and back in 15 minutes. Her speed on the way to the grazing land is 5 m/s, and on the way back her speed is 4 m/s. What is the distance to the grazing land?

**A5** Andrew and Bob each start with the 3-digit number 888 which is clearly divisible by 8. Andrew changes two of its digits in order to get as large a 3-digit number as he can which is still divisible by 8. Bob also changes two of the digits of 888 in order to get as small a 3-digit number as he can which is still divisible by 8. What is the difference between their two results? (None of the 3-digit numbers is allowed to begin with 0.)

## Section B

Answer each question on a separate sheet of A4 paper. Do not hand in rough working.

Try to finish whole questions even if you cannot do many: few candidates will do all five questions.

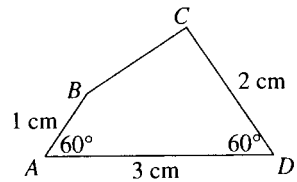
You should give full solutions, including clear mathematical explanations, and express all calculations and answers as exact numbers such as  $4\pi$ ,  $2 + \sqrt{7}$ . Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

- B1** A rectangular floor, which measures  $x$  feet by  $y$  feet, is covered in square tiles which are each 1 foot by 1 foot. All of the tiles on the perimeter of the rectangle are coloured blue, while all of the tiles in the interior of the rectangle are coloured yellow. There are three times as many yellow tiles on the floor as blue tiles.

- (a) Show that  $x$  and  $y$  satisfy the equation  $(x - 8)(y - 8) = 48$ .  
 (b) Hence find all possible values for the area of the floor.

- B2** In quadrilateral  $ABCD$ ,  $AB = 1$  cm,  $CD = 2$  cm,  $AD = 3$  cm and  $\angle BAD = \angle CDA = 60^\circ$ .

Calculate the length of side  $BC$ .



- B3** This question concerns sequences whose first term is neither 0 nor 1 and which are formed in the following way:

each term after the first term is equal to  $1 - (1 \div \text{the previous term})$ . For example, if the first term is 5 then the second term is  $1 - (1 \div 5) = \frac{4}{5}$ ; the third term is  $1 - (1 \div \frac{4}{5}) = -\frac{1}{4}$  and so on.

- (a) Find the first six terms of the sequence whose first term is 3.  
 (b) What are the first six terms of the sequence whose first term is  $x$ ?  
 (You should try to simplify expressions as much as possible.)  
 (c) What is the product of the first 100 terms of the sequence whose first term is  $x$ ?
- B4** In my town, 10% of the dogs think they are cats and 10% of the cats think they are dogs. All the other cats and dogs are perfectly normal. When all the cats and dogs in my town were rounded up and subjected to a rigorous test, 20% of the animals thought they were cats. What percentage of them really were cats?

- B5** A rectangular piece of paper  $ABCD$ , in which  $AB = 6$  cm and  $BC = 8$  cm, is folded once so that  $B$  folds exactly onto  $D$  and the folded paper is then laid flat on a table.

What area of the table is then covered by the paper?

