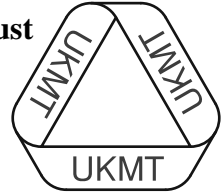


The United Kingdom Mathematics Trust



**Intermediate Mathematical Olympiad and Kangaroo
(IMOK)**

Olympiad Cayley Paper

Thursday 20th March 2014

All candidates must be in *School Year 9 or below* (England and Wales), *S2 or below* (Scotland), or *School Year 10 or below* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.
You may wish to work in rough first, then set out your final solution with clear explanations and proofs.
Do not hand in rough work.
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π , fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
Do not hurry, but spend time working carefully on one question before attempting another. Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.

Enquiries should be sent to: Maths Challenges Office,

School of Maths Satellite, University of Leeds, Leeds, LS2 9JT.

(Tel. 0113 343 2339)

<http://www.ukmt.org.uk>

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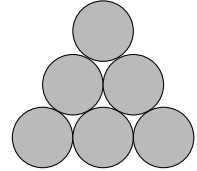
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- *Incomplete or poorly presented solutions will not receive full marks.*

- ***Do not hand in rough work.***

1. The two-digit integer '19' is equal to the product of its digits (1×9) plus the sum of its digits ($1 + 9$).

Find all two-digit integers with this property.

2. Six pool balls numbered 1–6 are to be arranged in a triangle, as shown. After three balls are placed in the bottom row, each of the remaining balls is placed so that its number is the difference of the two below it.



Which balls can land up at the top of the triangle?

3. Rachel gave half of her money to Howard. Then Howard gave a third of all his money to Rachel. They each ended up with the same amount of money.

Find the ratio

amount that Rachel started with : amount that Howard started with.

4. The square $ABIJ$ lies *inside* the regular octagon $ABCDEFGH$. The sides of the octagon have length 1.

Prove that $CJ = \sqrt{3}$.

5. Four types of rectangular tile have sizes $300 \text{ mm} \times 300 \text{ mm}$, $300 \text{ mm} \times 600 \text{ mm}$, $600 \text{ mm} \times 600 \text{ mm}$ and $600 \text{ mm} \times 900 \text{ mm}$. Equal numbers of each type of tile are used, without overlaps, to make a square.

What is the smallest square that can be made?

6. A couple own a circular piece of land that has area 2500 m^2 . The land is divided into four plots by two perpendicular chords that intersect at X . Their rectangular house H has diagonally opposite corners at X and at the centre of the circle O , as shown. The two plots A and B have a combined area of 1000 m^2 .

What is the area occupied by the house?

