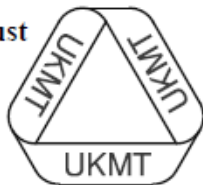


The United Kingdom Mathematics Trust



**Intermediate Mathematical Olympiad and Kangaroo  
(IMOK)**

**Olympiad Cayley Paper**

Thursday 15th March 2012

All candidates must be in *School Year 9 or below* (England and Wales), *S2 or below* (Scotland), or *School Year 10 or below* (Northern Ireland).

**READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING**

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**  
Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.  
You may wish to work in rough first, then set out your final solution with clear explanations and proofs.  
*Do not hand in rough work.*
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.  
Do not hurry, but spend time working carefully on one question before attempting another. Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

**DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!**

The United Kingdom Mathematics Trust is a Registered Charity.

*Enquiries should be sent to: Maths Challenges Office,  
School of Maths Satellite, University of Leeds, Leeds, LS2 9JT.  
(Tel. 0113 343 2339)*

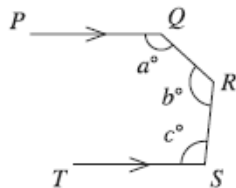
*<http://www.ukmt.org.uk>*

1. The digits  $p, q, r, s$  and  $t$  are all different.

What is the smallest five-digit integer ' $pqrst$ ' that is divisible by 1, 2, 3, 4 and 5?

2. In the diagram,  $PQ$  and  $TS$  are parallel.

Prove that  $a + b + c = 360$ .

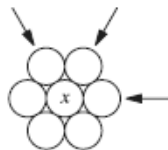


3. Three loaves of bread, five cartons of milk and four jars of jam cost £10.10.  
Five loaves of bread, nine cartons of milk and seven jars of jam cost £18.20.  
How much does it cost to buy one loaf of bread, one carton of milk and one jar of jam?

4. The diagram shows seven circles. Each of the three arrows indicates a 'line of three circles'.

The digits from 1 to 7 inclusive are to be placed in the circles, one per circle, so that the sum of the digits in each of the three indicated 'lines of three circles' is the same.

Find all possible values of  $x$ .



5. Every cell of the following crossnumber is to contain a single digit. No clue has an answer starting with zero.

Prove that there is exactly one solution to the crossnumber.

*Across*

2 Sum of the digits of 2 Down.

4 Prime.

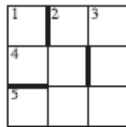
5  $1 \text{ Down} + 2 \text{ Across} + 3 \text{ Down}$ .

*Down*

1 Product of two primes.

2 Multiple of 99.

3 Square of 4 Across.



6. The diagram shows a symmetrical four-pointed star. Four vertices of the star form a square and the other four vertices lie on a circle. The square has sides of length  $2a$  cm. The shaded area is one third of the area of the square.

What is the radius of the circle?

