The United Kingdom Mathematics Trust

Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Cayley Paper

Thursday 17th March 2011

All candidates must be in *School Year 9 or below* (England and Wales), *S2 or below* (Scotland), or *School Year 10 or below* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.** Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.
   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

   Do not hand in rough work.
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.
   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.
Enquiries should be sent to: Maths Challenges Office,
School of Maths Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

- *Do not hurry, but spend time working carefully on one question before attempting another.*
- *Try to finish whole questions even if you cannot do many.*
- *You will have done well if you hand in full solutions to two or more questions.*

- *Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.*

- *Give full written solutions, including mathematical reasons as to why your method is correct.*
- *Just stating an answer, even a correct one, will earn you very few marks.*
- *Incomplete or poorly presented solutions will not receive full marks.*

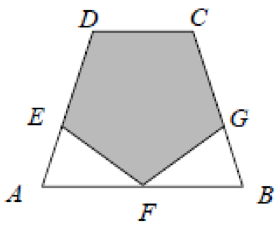
- ***Do not hand in rough work.***

1. A palindromic number is one which reads the same when its digits are reversed, for example 23832.

   What is the largest six-digit palindromic number which is exactly divisible by 15?

2. The diagram shows a regular pentagon *CDEFG* inside a trapezium *ABCD*.

   Prove that $AB = 2 \times CD$.

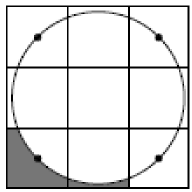


3. At dinner on a camping expedition, each tin of soup was shared between 2 campers, each tin of meatballs was shared between 3 campers and each tin of chocolate pudding was shared between 4 campers. Each camper had all three courses and all tins were emptied. The camp leader opened 156 tins in total.

   How many campers were on the expedition?

4. The diagram shows nine 1 cm × 1 cm squares and a circle. The circle passes through the centres of the four corner squares.

   What is the area of the shaded region—inside two squares but outside the circle?



5. Solve the equation $5a - ab = 9b^2$, where a and b are positive integers.

6. A bug starts in the small triangle *T* at the top of the diagram. She is allowed to eat through a neighbouring edge to get to a neighbouring small triangle. So at first there is only one possible move (downwards), and only one way to reach this new triangle.

   (a) How many triangles, including *T* and *B*, must the bug visit if she is to reach the small triangle *B* at the bottom using a route that is as short as possible?

   (b) How many different ways are there for the bug to reach *B* from *T* by a route of this shortest possible length?