The United Kingdom Mathematics Trust

Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Cayley/Hamilton/Maclaurin Papers

Thursday 18th March 2010

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.
   You may wish to work in rough first, then set out your final solution with clear explanations and proofs. ***Do not hand in rough work.***
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.
   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another. Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.

Enquiries should be sent to: Maths Challenges Office,
School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

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- ***Do not hand in rough work.***

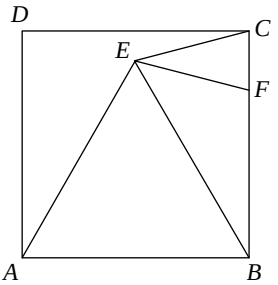
Olympiad Cayley Paper

All candidates must be in *School Year 9 or below* (England and Wales), *S2 or below* (Scotland), or *School Year 10 or below* (Northern Ireland).

1. The sum of three positive integers is 11 and the sum of the cubes of these numbers is 251.

   Find all such triples of numbers.

2. The diagram shows a square *ABCD* and an equilateral triangle *ABE*. The point *F* lies on *BC* so that *EC* = *EF*.
   Calculate the angle *BEF*.



3. Find all possible solutions to the 'word sum' on the right.
   Each letter stands for one of the digits 0−9 and has the same meaning each time it occurs. Different letters stand for different digits. No number starts with a zero.

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  O D D
+ O D D
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E V E N
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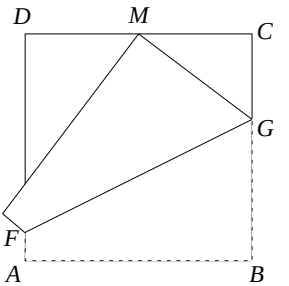
4. Walking at constant speeds, Eoin and his sister Angharad take 40 minutes and 60 minutes respectively to walk to the nearest town.
   Yesterday, Eoin left home 12 minutes after Angharad. How long was it before he caught up with her?

5. A square sheet of paper *ABCD* is folded along *FG*, as shown, so that the corner *B* is folded onto the midpoint *M* of *CD*.
   Prove that the sides of triangle *GCM* have lengths in the ratio 3 : 4 : 5.



6. A 'qprime' number is a positive integer which is the product of exactly two different primes, that is, one of the form $q \times p$, where q and p are prime and $q \neq p$.
   What is the length of the longest possible sequence of *consecutive* integers all of which are qprime numbers?