



The United Kingdom Mathematics Trust



**Intermediate Mathematical Olympiad and Kangaroo
(IMOK)**

Olympiad Cayley Paper

Thursday 19th March 2009

All candidates must be in *School Year 9 or below (England and Wales), S2 or below (Scotland), or School Year 10 or below (Northern Ireland).*

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.
You may wish to work in rough first, then set out your final solution with clear explanations and proofs.
Do not hand in rough work.
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π , fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
Do not hurry, but spend time working carefully on one question before attempting another. Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.

Enquiries should be sent to: Maths Challenges Office,

School of Mathematics, University of Leeds, Leeds, LS2 9JT.

(Tel. 0113 343 2339)

<http://www.ukmt.org.uk>

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1. An aquarium contains 280 tropical fish of various kinds. If 60 more clownfish were added to the aquarium, the proportion of clownfish would be doubled. How many clownfish are in the aquarium?

2. The boundary of the shaded figure consists of four semicircular arcs whose radii are all different. The centre of each arc lies on the line AB , which is 10 cm long.



What is the length of the perimeter of the figure?

3. Two different rectangles are placed together, edge-to-edge, to form a large rectangle. The length of the perimeter of the large rectangle is $\frac{2}{3}$ of the total perimeter of the original two rectangles.

Prove that the final rectangle is in fact a square.

4. In the rectangle $ABCD$, the side AB has length $\sqrt{2}$ and the side AD has length 1. Let the circle with centre B and passing through C meet AB at X .

Find $\angle ADX$ (in degrees).

5. Two candles are the same height. The first takes 10 hours to burn completely whilst the second takes 8 hours to burn completely.

Both candles are lit at midday. At what time is the height of the first candle twice the height of the second candle?

6. Teams A, B, C and D competed against each other once. The results table was as follows:

Team	Win	Draw	Loss	Goals for	Goals against
A	3	0	0	5	1
B	1	1	1	2	2
C	0	2	1	5	6
D	0	1	2	3	6

- (a) Find (with proof) which team won in each of the six matches.
 (b) Find (with proof) the scores in each of the six matches.