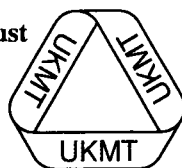


The United Kingdom Mathematics Trust



**Intermediate Mathematical Olympiad and Kangaroo  
(IMOK)**

**Olympiad Cayley/Hamilton/Maclaurin Papers**

Thursday 13th March 2008

**READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING**

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**  
Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.  
You may wish to work in rough first, then set out your final solution with clear explanations and proofs.  
***Do not hand in rough work.***
5. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like  $\pi$ , fractions, or square roots if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.  
Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.  
Do not hurry, but spend time working carefully on one question before attempting another. Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

**DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE  
INVIGILATOR TO DO SO!**

The United Kingdom Mathematics Trust is a Registered Charity.

*Enquiries should be sent to: Maths Challenges Office,*

*School of Mathematics, University of Leeds, Leeds, LS2 9JT.*

*(Tel. 0113 343 2339)*

*<http://www.ukmt.org.uk>*

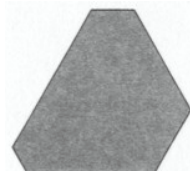
- *Do not hurry, but spend time working carefully on one question before attempting another.*
- *Try to finish whole questions even if you cannot do many.*
- *You will have done well if you hand in full solutions to two or more questions.*
- *Answers must be FULLY SIMPLIFIED, and EXACT using symbols like  $\pi$ , fractions, or square roots if appropriate, but NOT decimal approximations.*
- *Give full written solutions, including mathematical reasons as to why your method is correct.*
- *Just stating an answer, even a correct one, will earn you very few marks.*
- *Incomplete or poorly presented solutions will not receive full marks.*
- ***Do not hand in rough work***

## Olympiad Cayley Paper

**All candidates must be in *School Year 9 or below* (England and Wales), *S2 or below* (Scotland), or *School Year 10 or below* (Northern Ireland).**

1. How many four-digit multiples of 9 consist of four different odd digits?

2. A hexagon is made by cutting a small equilateral triangle from each corner of a larger equilateral triangle. The sides of the smaller triangles have lengths 1, 2 and 3 units. The lengths of the perimeters of the hexagon and the original triangle are in the ratio 5 : 7.



What fraction of the area of the original triangle remains?

3. In the rectangle  $ABCD$  the midpoint of  $AB$  is  $M$  and  $AB : AD = 2 : 1$ . The point  $X$  is such that triangle  $MDX$  is equilateral, with  $X$  and  $A$  lying on opposite sides of the line  $MD$ . Find the value of  $\angle XCD$ .

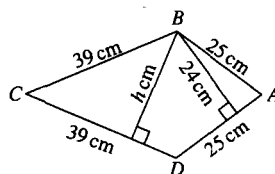
4. The number  $N$  is the product of the first 99 positive integers. The number  $M$  is the product of the first 99 positive integers after each has been reversed. That is, for example, the reverse of 8 is 8; of 17 is 71; and of 20 is 02.

Find the exact value of  $N \div M$ .

5. A kite has sides  $AB$  and  $AD$  of length 25 cm and sides  $CB$  and  $CD$  of length 39 cm. The perpendicular distance from  $B$  to  $AD$  is 24 cm.

The perpendicular distance from  $B$  to  $CD$  is  $h$  cm.

Find the value of  $h$ .



6. A regular tetrahedron  $ABCD$  has edges of length 2 units. The midpoint of the edge  $AB$  is  $M$  and the midpoint of the edge  $CD$  is  $N$ .

Find the exact length of the segment  $MN$ .