The United Kingdom Mathematics Trust

Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Cayley Paper

Thursday 15th March 2007

All candidates must be in *School Year 9 or below* (England and Wales), *S2 or below* (Scotland), or *School Year 10 or below* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.
   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

   Do not hand in rough work.
5. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.
   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.
Enquiries should be sent to: Maths Challenges Office,
School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

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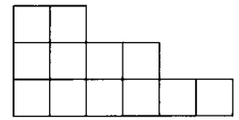
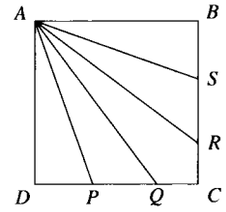
- ***Do not hand in rough work***

1. Four copies of the polygon shown are fitted together (without gaps or overlaps) to form a rectangle.
How many different rectangles are possible?



2. Before the last of a series of tests, Sam calculated that a mark of 17 would enable her to average 80 over the series, but that a mark of 92 would raise her average mark over the series to 85.
How many tests were there in the series?

3. The diagram shows a square $ABCD$ of side 10 units. Line segments AP, AQ, AR and AS divide the square into five regions of equal area, as shown.
Calculate the length of QR.



4. How many right-angled triangles can be made by joining three vertices of a cube?

5. In a quadrilateral $ABCD$, $AB = BC$, $\angle BAC = 60°$, $\angle CAD = 40°$, AC and BD cross at X and $\angle BXC = 100°$.
Calculate $\angle BDC$.

6. (a) You are told that one of the integers in a list of distinct positive integers is 97 and that their average value is 47. If the sum of all the integers in the list is 329, what is the largest possible value for a number in the list?
(b) Suppose the sum of all the numbers in the list can take any value. What would the largest possible number in the list be then?