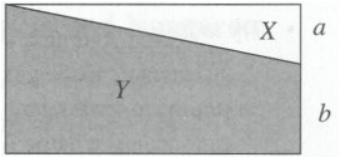
Olympiad Cayley Paper

All candidates must be in *School Year 9 or below* (England and Wales), *S2 or below* (Scotland), or *School Year 10 or below* (Northern Ireland).

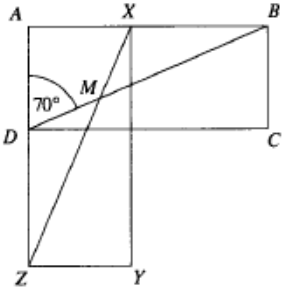
1. A rectangular piece of paper is cut into two pieces by a straight line passing through one corner, as shown.
   Given that area X : area $Y = 2 : 7$, what is the value of the ratio $a : b$?



2. Show that there are no solutions to this "letter sum".
   [Each letter stands for one of the digits 0–9; different letters stand for different digits; no number begins with the digit 0.]

$$\begin{array}{r} \text{SEVEN} \\ +\ \text{ONE} \\ \hline \text{EIGHT} \end{array}$$

3. In the diagram, rectangles $ABCD$ and $AZYX$ are congruent, and angle $ADB = 70°$.
   Find angle BMX.



4. Find the positive integer whose value is increased by 518059 when the digit 5 is placed at each end of the number.

5. Mij the magician has a large bag of red balls and a large bag of green balls. Mij wanders round the audience selecting volunteers, asking each volunteer to remove two balls, one from each bag, until $\frac{2}{5}$ of the red balls and $\frac{3}{7}$ of the green balls have been removed. The balls remaining in the bags are then emptied into a bucket.
   What fraction of all the balls does the bucket contain?

6. A mathematician has a full one-litre bottle of concentrated orange squash, a large container and a tap. He first pours half of the bottle of orange squash into the container. Then he fills the bottle from the tap, shakes well, and pours half of the resulting mixture into the container. He then repeats this step over and over again: filling the bottle from the tap each time, shaking the mixture well, and then pouring half of the contents into the container.
   Suppose that on the final occasion he fills the bottle from the tap and empties it completely into the container. How many times has he filled the bottle from the tap if the final mixture consists of 10% orange squash concentrate?