

## Olympiad Cayley Paper

All candidates must be in *School Year 9 or below (England and Wales), S2 or below (Scotland), or School Year 10 or below (Northern Ireland).*

1. The edge length, in centimetres, of a solid wooden cube is a whole number greater than two. The outside of the cube is painted blue and the cube is then cut into small cubes whose edge length is 1 cm. The number of small cubes with exactly one blue face is ten times the number of small cubes with exactly two blue faces.  
Find the edge length of the original cube.

2. When two congruent isosceles triangles are joined to form a parallelogram, as shown in the first diagram, the perimeter of the parallelogram is 3 cm longer than the perimeter of one of the triangles.



When the same two triangles are joined to form a rhombus, as shown in the second diagram, the perimeter of the rhombus is 7 cm longer than the perimeter of one of the triangles.



What is the perimeter of one of the triangles?

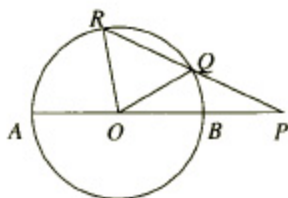
3. In triangle  $ABC$ , angle  $B$  is a right angle and  $X$  is the point on  $BC$  so that  $BX : XC = 5 : 4$ . Also, the length of  $AB$  is three times the length of  $CX$  and the area of triangle  $CXA$  is  $54 \text{ cm}^2$ . Calculate the length of the perimeter of triangle  $CXA$ .



4. The five-digit number 'a679b', where  $a$  and  $b$  are digits, is divisible by 36. Find all possible such five-digit numbers.

5. In the diagram,  $O$  is the centre of the circle and the straight lines  $AOBP$  and  $RQP$  meet at  $P$ . The length of  $PQ$  is equal to the radius of the circle. Prove that

$$\angle AOR = 3 \times \angle BOQ.$$



6. If you have an endless supply of  $3 \times 2$  rectangular tiles, you can place 100 tiles end to end to tile a  $300 \times 2$  rectangle. Similarly, you can put  $k$  tiles side by side to tile a  $3k \times 2$  rectangle.

Find the values of the integers  $k$  and  $m$  for which it is possible to tile a  $6k \times m$  rectangle with  $3 \times 2$  tiles.