The United Kingdom Mathematics Trust

Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Cayley Paper

Thursday 20th March 2003

All candidates must be in *School Year 9 or below* (England and Wales), *S2 or below* (Scotland), or *School Year 10 or below* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.
3. For questions in Section A *only the answer is required.* Enter each answer neatly in the relevant box on the Cover Sheet.

   For questions in Section B start each question on a fresh A4 sheet and give *full written solutions*, including clear mathematical explanations as to why your method is correct.

   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.

   Do not hand in rough work.
4. Questions A1-A5 are relatively short questions. Try to complete Section A within the first 20 minutes so as to allow sufficient time for Section B.
5. Questions B1-B5 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
6. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
7. Numerical answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

Section A

Write your answers in the boxes provided on the Cover Sheet. Do not hand in your working.
You should aim to spend no more than 20 minutes on this section.

A1 There are 17 trees along the road from Basil's home to a swimming pool. On his way to and from a swim Basil marks some trees with a red stripe. On his way to the pool he marks the first tree, the third tree, the fifth tree and so on. On his way back again, he marks the first tree he comes to, the fourth tree, the seventh tree and so on, missing out two trees each time. By the time he gets home, how many trees have no mark?

A2 A straight line is drawn across a 4×4 grid (like a chessboard). What is the greatest number of 1×1 squares which can be divided into two by the line?

A3 There used to be 5 parrots in my cage. Their average value was €6000. One day while I was cleaning out the cage the most beautiful parrot flew away. The average value of the remaining four parrots was €5000. What was the value of the parrot that escaped?

A4 Start with a positive integer with 2 digits. Crossing out the units digit gives a new single digit number. If you multiply this new number by an integer x you get the original number back. What is the greatest possible value of x?

A5 Mike has 42 identical cubes, each with edges of length 1 cm. He used all the cubes to construct a cuboid. The perimeter of the base of that cuboid was 18 cm. What was its height?

Section B

Answer each question on a separate sheet of A4 paper. Do not hand in rough working.

Try to finish whole questions even if you cannot do many: few candidates will do all five questions.

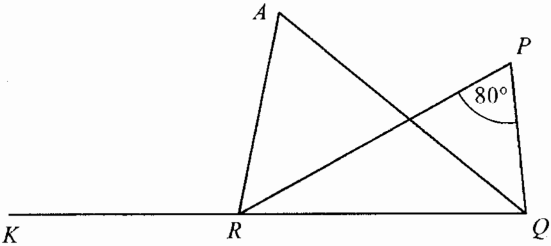
You should give full solutions, including clear mathematical explanations, and express all calculations and answers as exact numbers such as 4π, $2 + \sqrt{7}$. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

B1 In this multiplication sum, p, q, r and s stand for different digits. Find the digit which each letter represents, explaining how you know that you have found all possible solutions.

$$\begin{array}{r} p\ q \\ \times\ r\ q \\ \hline s\ s\ s \\ \hline \end{array}$$

B2 In the diagram (which is not to scale), AQ bisects $\angle PQR$, AR bisects $\angle KRP$ and $\angle RPQ = 80°$.

What is the size of $\angle RAQ$?

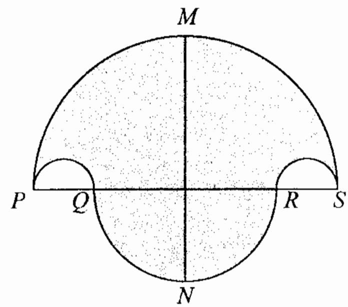


B3 At McBride Academy there are 300 children each of whom represents the school in both summer and winter sports. In summer, 60% of these play tennis and the other 40% play badminton. In winter they play hockey or swim but not both. 56% of the hockey players play tennis in summer and 30% of the tennis players swim.

How many both swim and play badminton?

B4 In the diagram, P, Q, R and S are four points on a line such that $PQ = RS$. Semicircles are drawn above the line with diameters PQ, RS and PS, and another semicircle with diameter QR is drawn below the line. The line MN is the line of symmetry of the figure.

Prove that the shaded area is equal to the area of the circle with diameter MN.



B5 The numbers 1 to 9 are to be placed in the circles in such a way that the sum of the four numbers along each side of the triangle has the same value, T say.

(a) Prove that $17 \leqslant T \leqslant 23$.

(b) Find a suitable arrangement of the numbers when $T = 23$.

(c) Show that when $T = 20$ then there are at most 8 *different* choices for the collection of three numbers which should be placed at the vertices of the triangle.